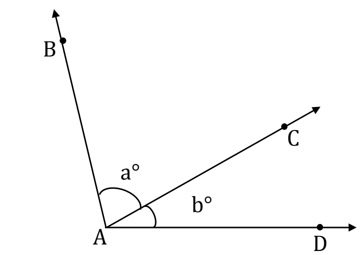
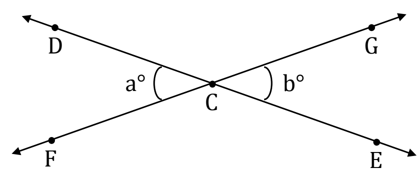
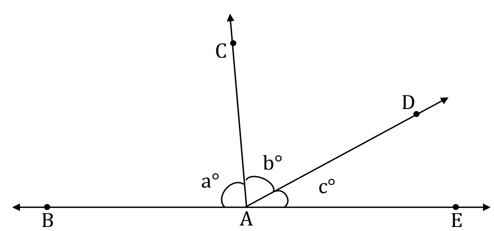
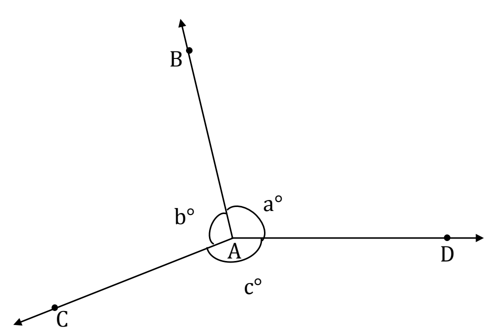
GEOMETRY

7 TH GRADE

Lesson 10 (from Module 3): Angle Problems and Solving Equations

Classwork

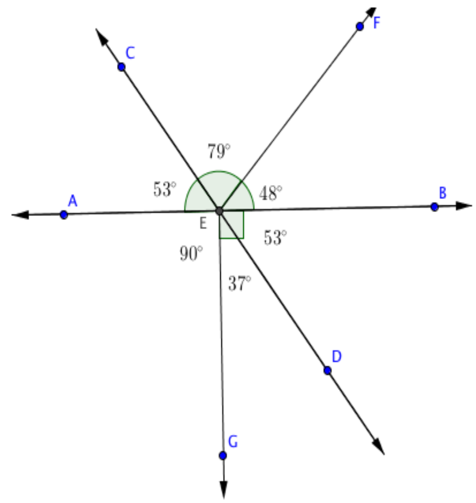
Angle Facts and Definitions

Name of Angle Relationship	Angle Fact	Diagram
Adjacent Angles		
Vertical Angles (vert. ∠s)		
Angles on a Line (∠s on a line)		
Angles at a Point (∠s at a point)		

Opening Exercise

Use the diagram to complete the chart.

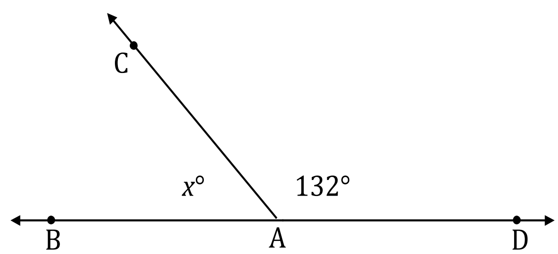
Name the Angles that are …	
Vertical	
Adjacent	
Angles on a line	
Angles at a point	



Example 1

Estimate the measurement of x. ______________

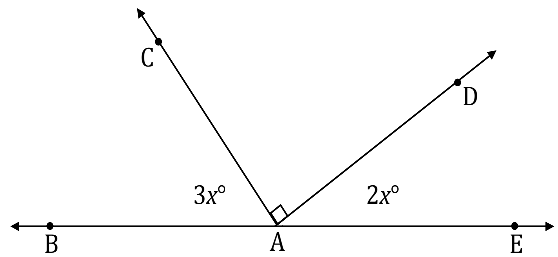
In a complete sentence, describe the angle relationship in the diagram.



Write an equation for the angle relationship shown in the figure and solve for x. Then find the measures of $\angle BAC$ and confirm your answers by measuring the angle with a protractor.

Exercise 1

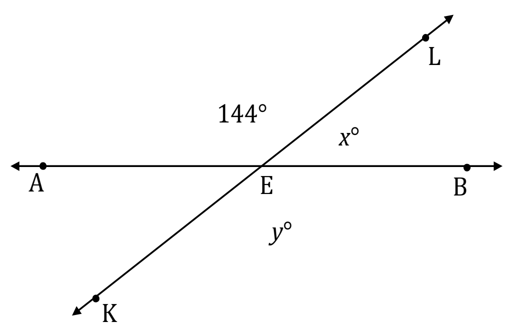
In a complete sentence, describe the angle relationship in the diagram.



Find the measurements of $\angle BAC$ and $\angle DAE$.

Example 2

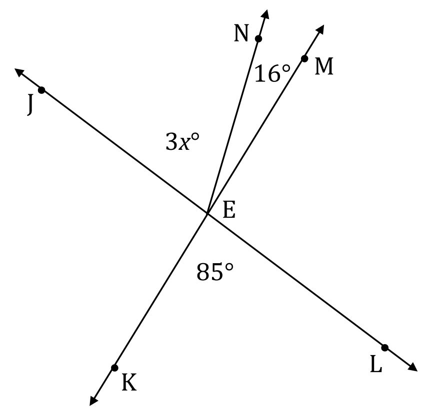
In a complete sentence, describe the angle relationship in the diagram.



Write an equation for the angle relationship shown in the figure and solve for x and y. Find the measurements of $\angle LEB$ and $\angle KEB$.

Exercise 2

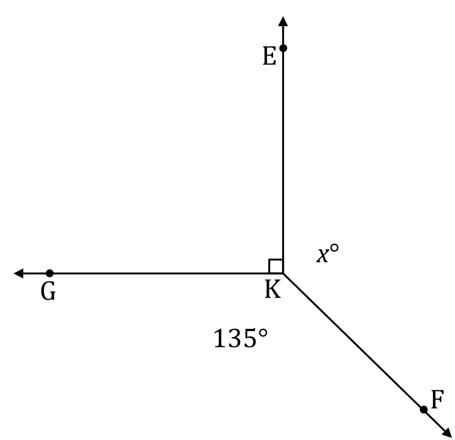
In a complete sentence, describe the angle relationships in the diagram.



Write an equation for the angle relationship shown in the figure and solve for x.

Example 3

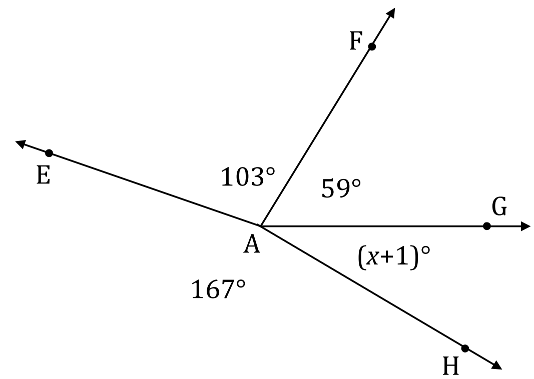
In a complete sentence, describe the angle relationships in the diagram.



Write an equation for the angle relationship shown in the figure and solve for x. Find the measurement of $\angle EKF$ and confirm your answers by measuring the angle with a protractor.

Exercise 3

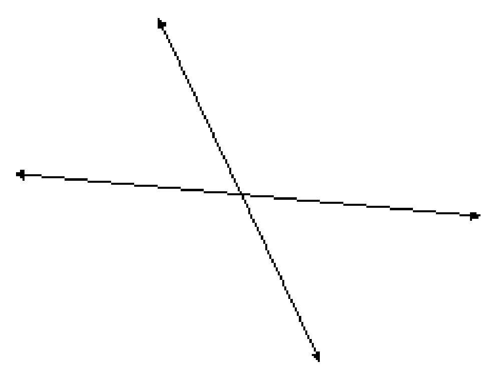
In a complete sentence, describe the angle relationships in the diagram.



Find the measurement of $\angle GAH$.

Example 4

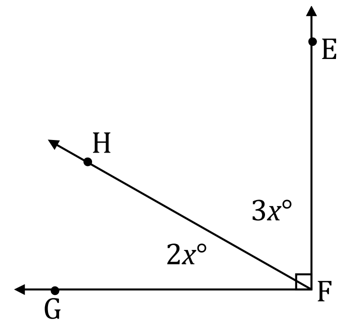
The following two lines intersect. The ratio of the measurements of the obtuse angle to the acute angle in any adjacent angle pair in this figure is $2:1$. In a complete sentence, describe the angle relationships in the diagram.



Label the diagram with expressions that describe this relationship. Write an equation that models the angle relationship and solve for x. Find the measurements of the acute and obtuse angles.

Exercise 4

The ratio of $\angle GFH$ to $\angle EFH$ is $2:3$. In a complete sentence, describe the angle relationships in the diagram.



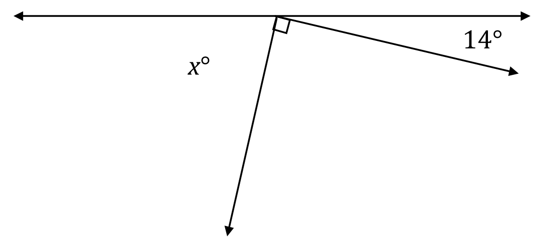
Find the measures of $\angle GFH$ and $\angle EFH$.

Lesson 11 (from Module 3): Angle Problems and Solving Equations

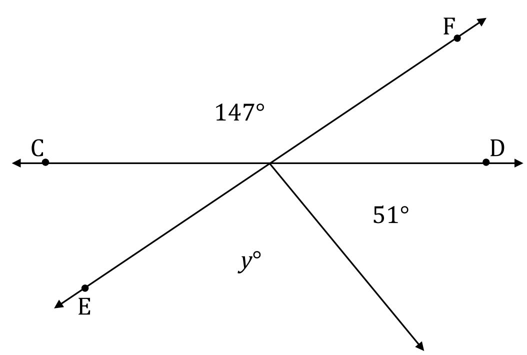
Classwork

Opening Exercise

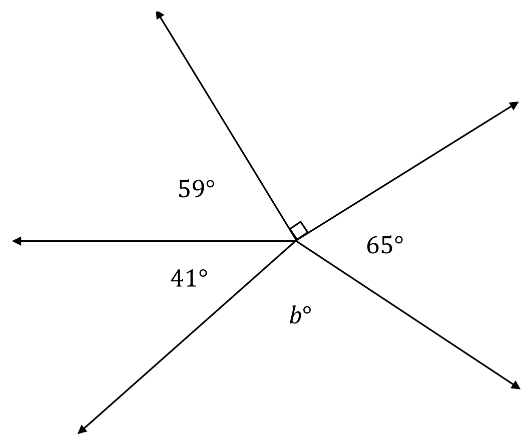
a. In a complete sentence, describe the angle relationship in the diagram. Write an equation for the angle relationship shown in the figure and solve for x. Confirm your answers by measuring the angle with a protractor.



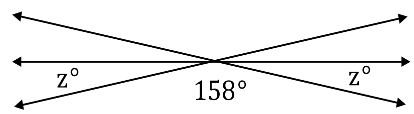
b. CD and EF are intersecting lines. In a complete sentence, describe the angle relationship in the diagram. Write an equation for the angle relationship shown in the figure and solve for y. Confirm your answers by measuring the angle with a protractor.



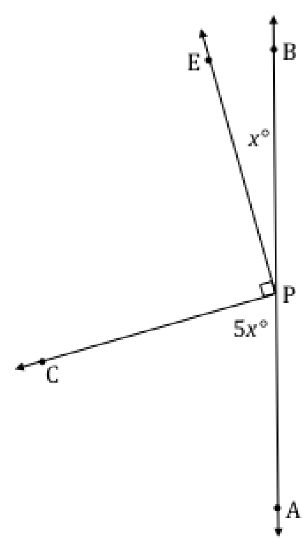
c. In a complete sentence, describe the angle relationship in the diagram. Write an equation for the angle relationship shown in the figure and solve for b. Confirm your answers by measuring the angle with a protractor.



d. The following figure shows three lines intersecting at a point. In a complete sentence, describe the angle relationship in the diagram. Write an equation for the angle relationship shown in the figure and solve for z. Confirm your answers by measuring the angle with a protractor.

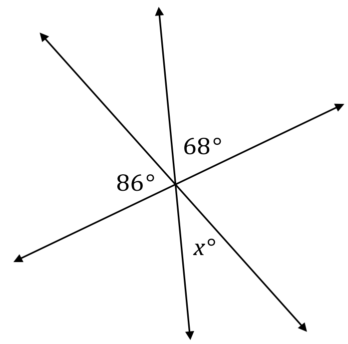


e. Write an equation for the angle relationship shown in the figure and solve for x. In a complete sentence, describe the angle relationship in the diagram. Find the measurements of $\angle EPB$ and $\angle CPA$. Confirm your answers by measuring the angle with a protractor.



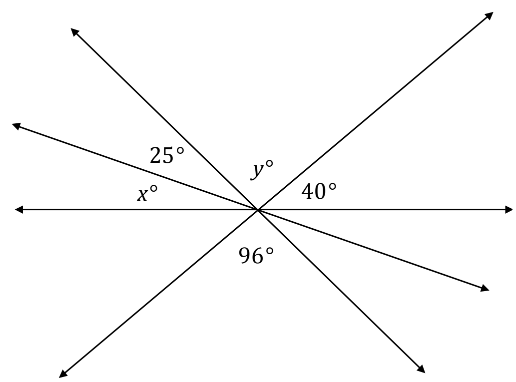
Example 1

The following figure shows three lines intersecting at a point. In a complete sentence, describe the angle relationship in the diagram. Write an equation for the angle relationship shown in the figure and solve for x. Confirm your answers by measuring the angle with a protractor.



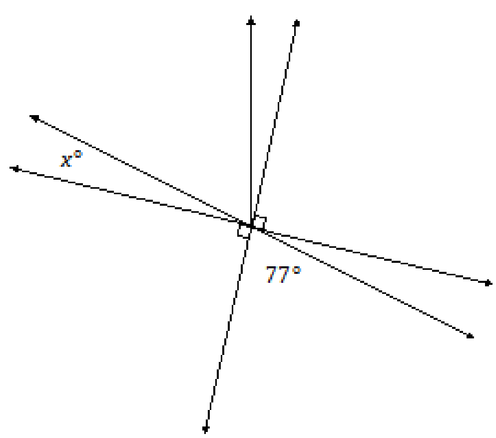
Exercise 1

The following figure shows four lines intersecting at a point. In a complete sentence, describe the angle relationships in the diagram. Write an equation for the angle relationship shown in the figure and solve for x and y. Confirm your answers by measuring the angle with a protractor.



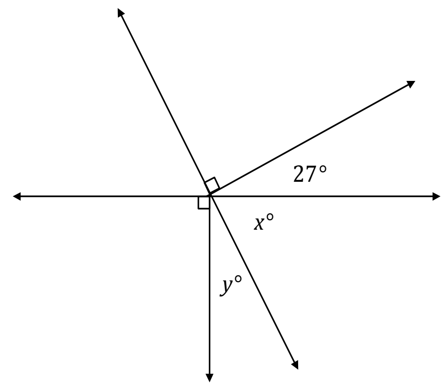
Example 2

In a complete sentence, describe the angle relationships in the diagram. You may label the diagram to help describe the angle relationships. Write an equation for the angle relationship shown in the figure and solve for x. Confirm your answers by measuring the angle with a protractor.



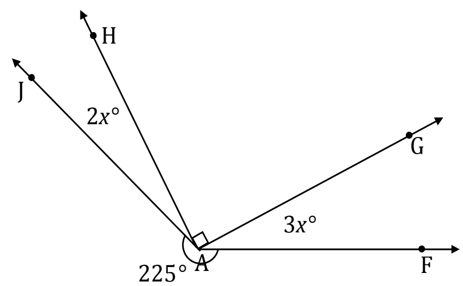
Exercise 2

In a complete sentence, describe the angle relationships in the diagram. Write an equation for the angle relationship shown in the figure and solve for x and y. Confirm your answers by measuring the angle with a protractor.



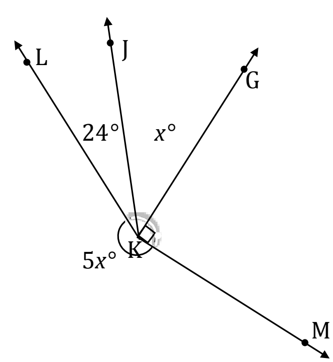
Example 3

In a complete sentence, describe the angle relationships in the diagram. Write an equation for the angle relationship shown in the figure and solve for x. Find the measures of $\angle JAH$ and $\angle GAF$. Confirm your answers by measuring the angle with a protractor.



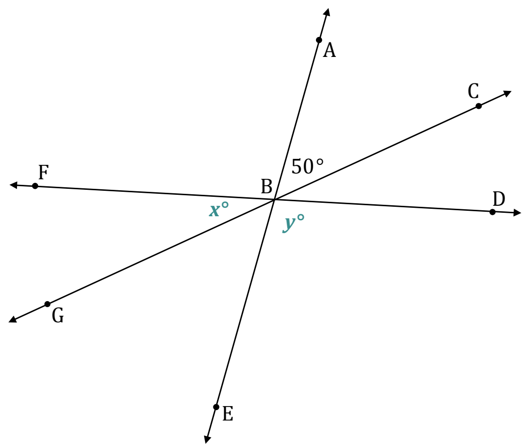
Exercise 3

In a complete sentence, describe the angle relationships in the diagram. Write an equation for the angle relationship shown in the figure and solve for x. Find the measures of $\angle JAH$ and $\angle GAF$. Confirm your answers by measuring the angle with a protractor.



Example 4

In the accompanying diagram, $\angle DBE$ is four times the measure of $\angle FBG$.



a. Label $\angle DBE$ as $y°$ and $\angle FBG$ as $x°$. Write an equation that describes the relationship between $\angle DBE$ and $\angle FBG$.

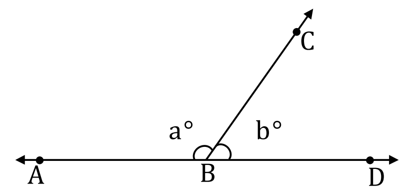
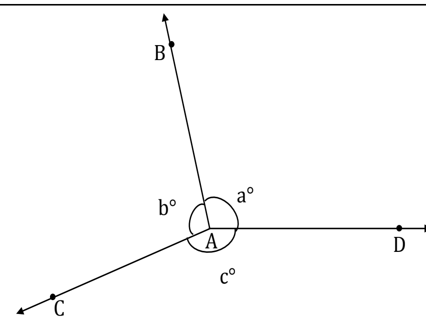
b. Find the value of x.

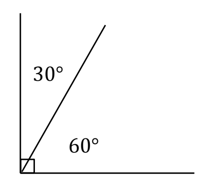
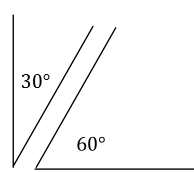
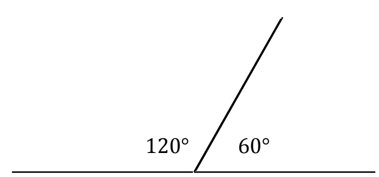
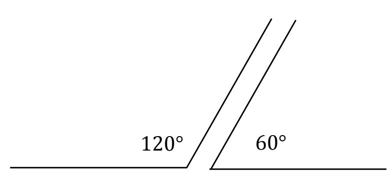
c. Find the measures of $\angle FBG$, $\angle CBD$, $\angle ABF$, $\angle GBE$, and $\angle DBE$.

d. What is the measure of $\angle ABG$? Identify the angle relationship used to get your answer.

Lesson 1: Complementary and Supplementary Angles

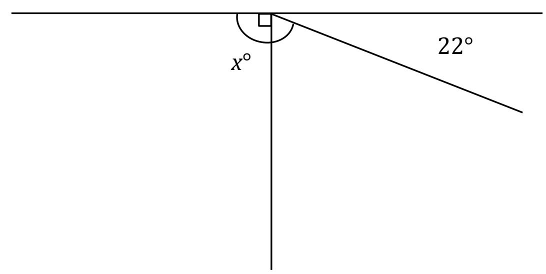
Classwork

Angles on a Line	If the vertex of a ray lies on a line but the ray is not contained in that line, then the sum of measurements of the two angles formed is $180°$.	
Angles at a Point	Suppose three or more rays with the same vertex separate the plane into angles with disjointed interiors. Then the sum of all the measurements of the angles is $360°$.	

Angle Relationship	Definition	Diagram
Complementary Angles		 
Supplementary Angles		 

Exercise 1

1. In a complete sentence, describe the relevant angle relationships in the diagram. Write an equation for the angle relationship shown in the figure and solve for x. Confirm your answers by measuring the angle with a protractor.



Example 1

The measures of two supplementary angles are in the ratio of $7:3$. Find the two angles.

Exercises 2–4

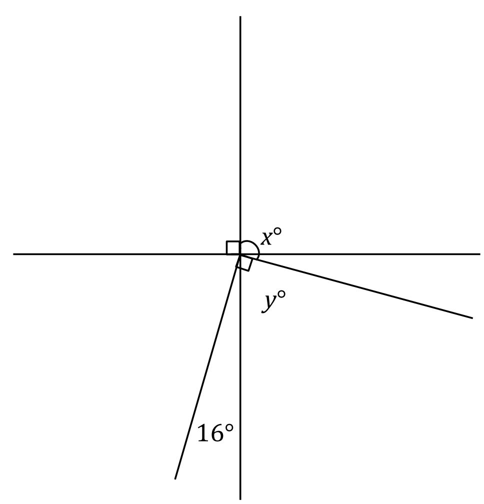
2. In a pair of complementary angles, the measurement of the larger angle is three times that of the smaller angle. Find the measurements of the two angles.

3. The measure of a supplement of an angle is $6°$ more than twice the measure of the angle. Find the two angles.

4. The measure of a complement of an angle is $32°$ more than three times the angle. Find the two angles.

Example 2

Two lines meet at the common vertex of two rays. Set up and solve an appropriate equation for x and y.

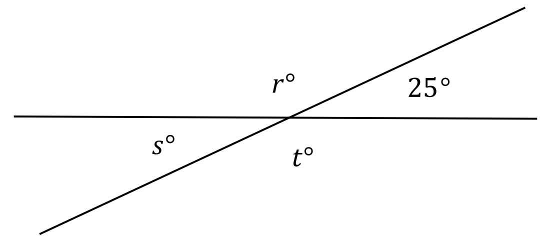


Lesson 2: Solving for Unknown Angles Using Equations

Classwork

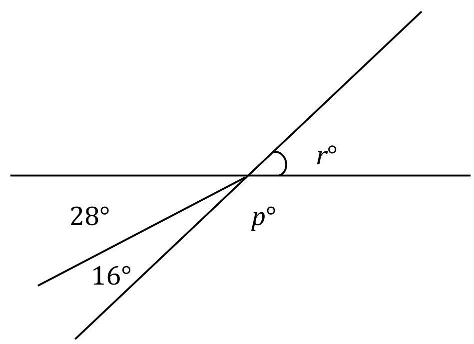
Opening Exercise

Two lines meet at a point. In a complete sentence, describe the relevant angle relationships in the diagram. Find the values of r, s, and t.



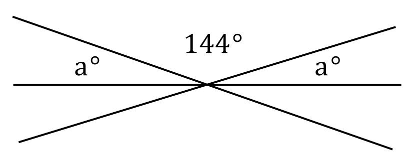
Example 1

Two lines meet at the vertex of a ray. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of p and r.



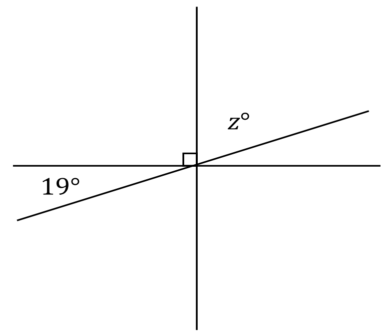
Exercise 1

Three lines meet at a point. In a complete sentence, describe the relevant angle relationship in the diagram. Set up and solve an equation to find the value of a.



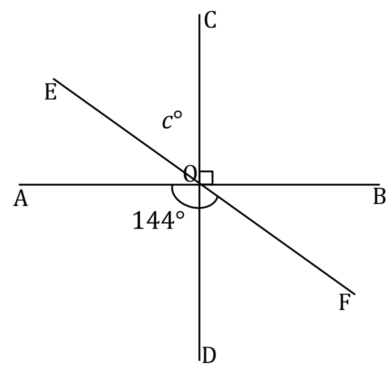
Example 2

Three lines meet at a point. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of z.



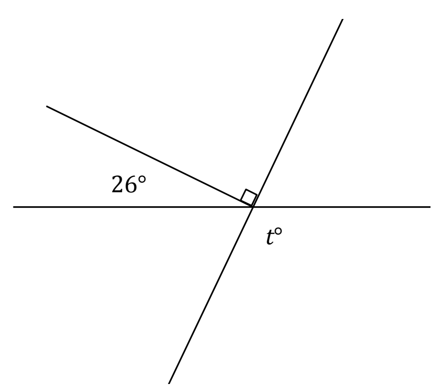
Exercise 2

Three lines meet at a point; $\angle AOF = 144°$. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to determine the value of c.



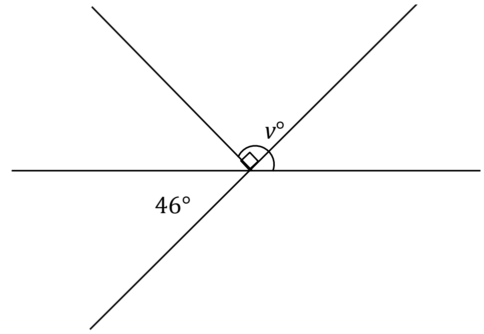
Example 3

Two lines meet at the vertex of a ray. The ray is perpendicular to one of the lines as shown. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of t.



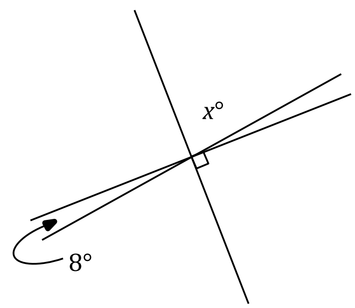
Exercise 3

Two lines meet at the vertex of a ray. The ray is perpendicular to one of the lines as shown. In a complete sentence, describe the relevant angle relationships in the diagram. You may add labels to the diagram to help with your description of the angle relationship. Set up and solve an equation to find the value of v.



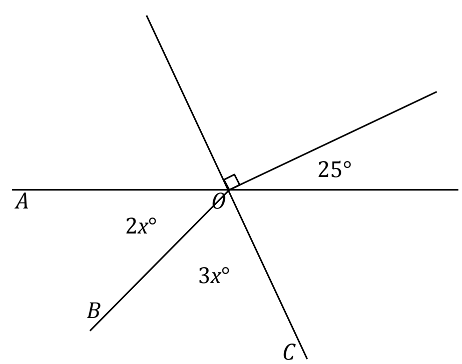
Example 4

Three lines meet at a point. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of x. Is your answer reasonable?



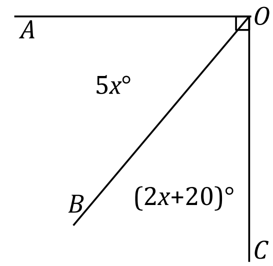
Exercise 4

Two lines meet at the common vertex of two rays. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of x. Find the measurements of $\angle AOB$ and$\angle BOC$.



Exercise 5

a. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of x. Find the measurements of $\angle AOB$ and $\angle BOC$.



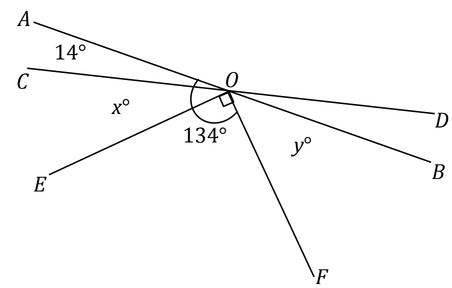
b. Katrina was solving the problem above and wrote the equation $7x + 20 = 90$. Then she rewrote this as $7x + 20 = 70 + 20$. Why did she rewrite the equation in this way? How does this help her to find the value of x?

Lesson 3: Solving for Unknown Angles using Equations

Classwork

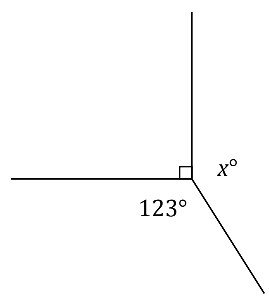
Opening Exercise

Two lines meet at the common vertex of two rays; the measurement of $\angle COF = 134°$. Set up and solve an equation to find the value of x and y.



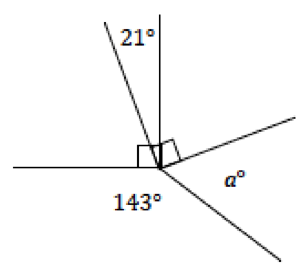
Example 1

Set up and solve an equation to find the value of x.



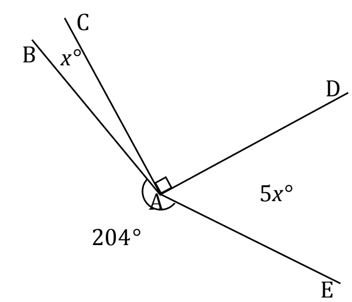
Exercise 1

Five rays meet at a common vertex. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of a.



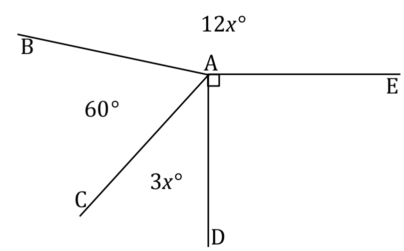
Example 2

Four rays meet at a common vertex. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of x. Find the measurements of angles $\angle BAC$ and $\angle DAE$.



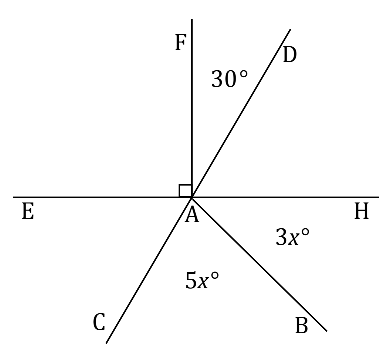
Exercise 2

Four rays meet at a common vertex. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of x. Find the measurement of $\angle CAD$.



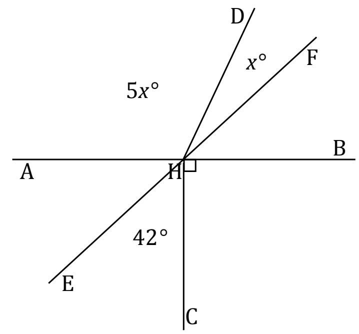
Example 3

Two lines meet at the common vertex of two rays. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of x. Find the measurements of angles $\angle BAC$ and $\angle BAH$.



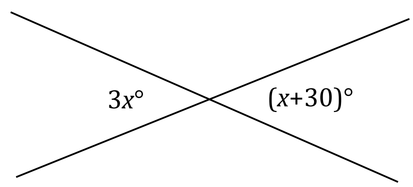
Exercise 3

Two lines meet at the common vertex of two rays. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of x. Find the measurements of angles $\angle DHF$ and $\angle AHD$.



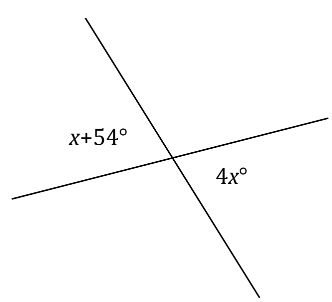
Example 4

Two lines meet at a point. Set up and solve an equation to find the value of x. Find the measurement of one of the vertical angles.



Exercise 4

Set up and solve an equation to find the value of x. Find the measurement of one of the vertical angles.



Lesson 4: Solving for Unknown Angles Using Equations

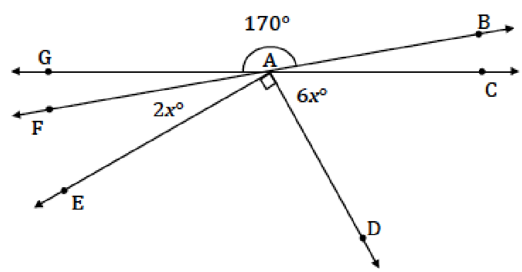
Classwork

Opening Exercise

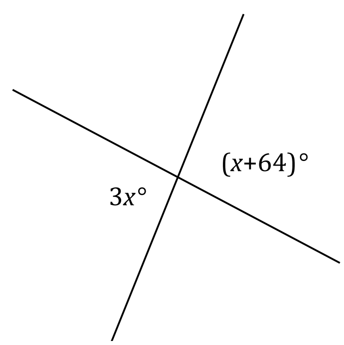
Four times the measurement of an angle is the complement of the angle. Find the measurement of the angle and its complement.

Example 1

Find the measurement of $\angle FAE$ and $\angle CAD$.

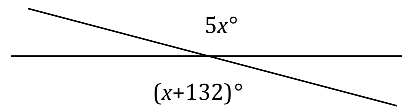


Two lines meet at a point. List the relevant angle relationship in the diagram. Set up and solve an equation to find the value of x. Find the measurement of one of the vertical angles.



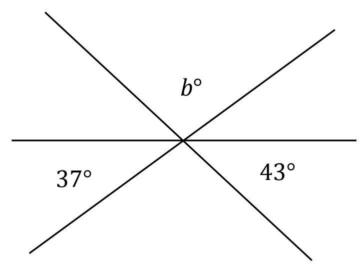
Exercise 1

Set up and solve an equation to find the value of x. List the relevant angle relationship in the diagram. Find the measurement of one of the vertical angles.



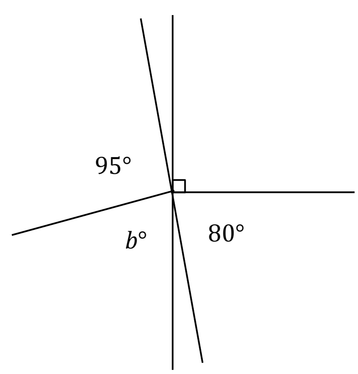
Example 2

Three lines meet at a point. List the relevant angle relationships in the diagram. Set up and solve an equation to find the value of b.



Exercise 2

Two lines meet at the common vertex of two rays. List the relevant angle relationships in the diagram. Set up and solve an equation to find the value of b.



Example 3

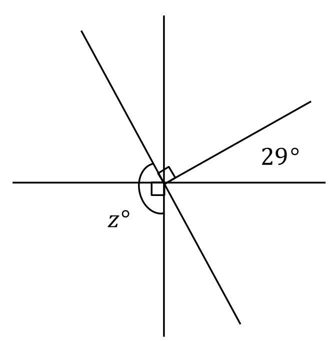
The measurement of an angle is $\frac{2}{3}$ the measurement of its supplement. Find the measurement of the angle.

Exercise 3

The measurement of an angle is $\frac{1}{4}$ the measurement of its complement. Find the measurement of the angle.

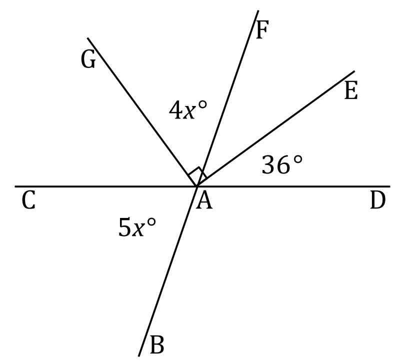
Example 4

Three lines meet at the common vertex of a ray. List the relevant angle relationships in the diagram. Set up and solve an equation to find the value of z.



Exercise 4

Two lines meet at the common vertex of two rays. Set up and solve an equation to find the value of x. Find the measurement of $\angle GAF$ and of $\angle BAC$.

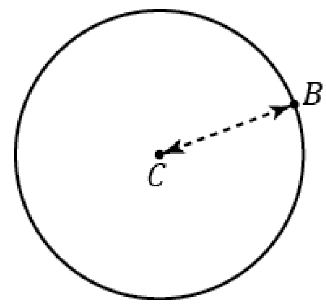


Lesson 16 (from Module 3): The Most Famous Ratio of All

Classwork

Opening Exercise

C is the *center* of the circle.
The distance between C and B is the *radius* of the circle.



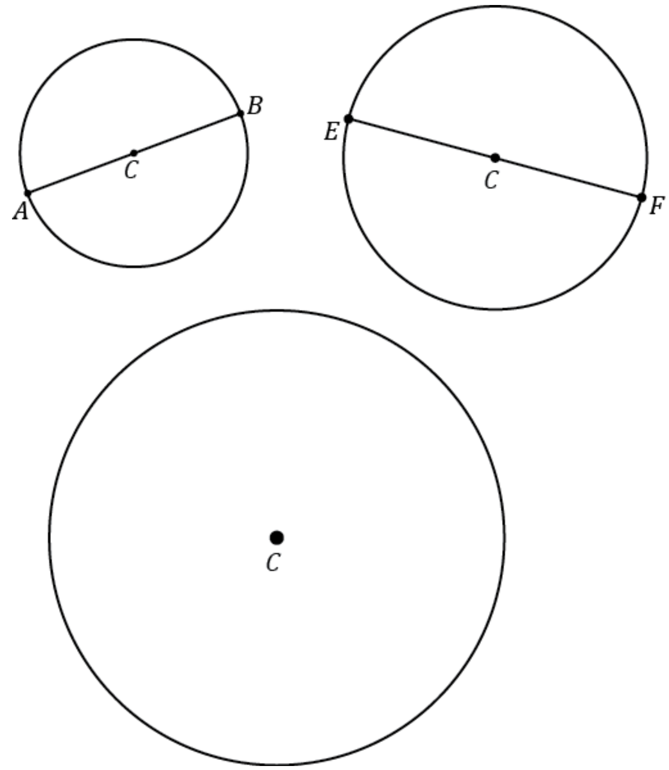
a. Write your own definition for the term circle.

b. Extend segment CB to a segment AB in part (a), where A is also a point on the circle.

The length of the segment AB is called the *diameter of the circle.*

c. The diameter is ________________________ as long as radius.

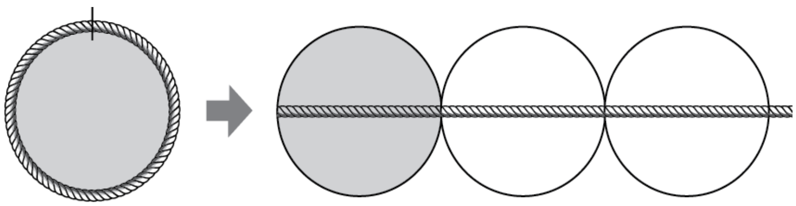
d. Measure the radius and diameter of each circle. The center of each circle is labeled C.



e. Draw a circle of radius 6 cm.

Mathematical Modeling Exercise

The ratio of the circumference to its diameter is always the same for any circle. The value of this ratio, $\frac{\text{Circumference}}{\text{Diameter}}$, is called the number *pi* and is represented by the symbol π.

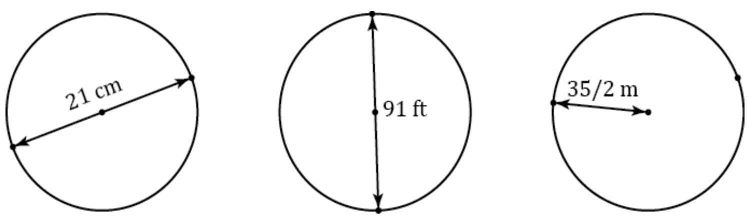


Since the circumference is a little greater than 3 times the diameter, π is a number that is a little greater than 3. Use the symbol π to represent this special number. Pi is a non-terminating, non-repeating decimal, and mathematicians use the symbol π or approximate representations as more convenient ways to represent pi.

- $\pi \approx 3.14$ or $\frac{22}{7}$.
- The ratios of the circumference to the diameter and $\pi : 1$ are equal.
- $\text{Circumference of a Circle} = \pi \times \text{Diameter}$.

Exercise 2

a. The following circles are not drawn to scale. Find the circumference of each circle. (Use $\frac{22}{7}$ as an approximation for π.)



b. The radius of a paper plate is 11.7 cm. Find the circumference to the nearest tenth. (Use 3.14 as an approximation for π.)

c. The radius of a paper plate is 11.7 cm. Find the circumference to the nearest hundredth. (Use the π button on your calculator as an approximation for π.)

d. A circle has a radius of r cm and a circumference of C cm. Write a formula that expresses the value of C in terms of r and π.

e. The figure below is in the shape of a semicircle. A semicircle is an arc that is "half" of a circle. Find the perimeter of the shape. (Use 3.14 for π.)

Lesson 17 (from Module 3): The Area of a Circle

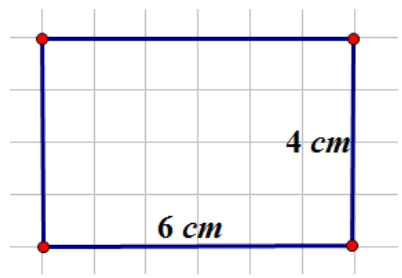
Classwork

Exercises 1–3

Solve the problem below individually. Explain your solution.

1. Find the radius of the following circle if the circumference is 37.68 inches. Use $\pi \approx 3.14$.

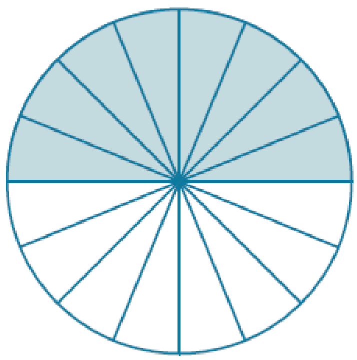
2. Determine the area of the rectangle below. Name two ways that can be used to find the area of the rectangle.



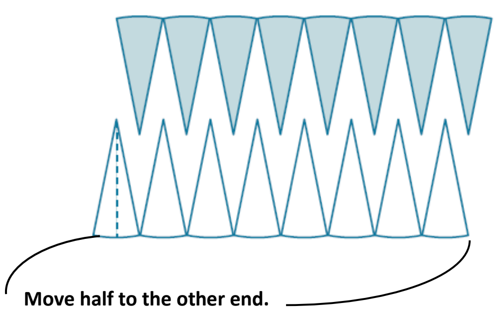
3. Find the length of a rectangle if the area is 27 cm^2 and the width is 3 cm.

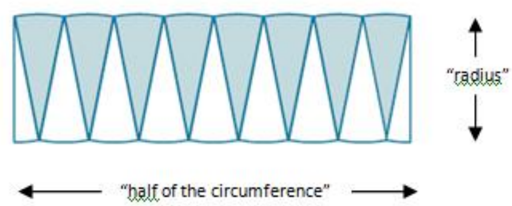
Exploratory Challenge

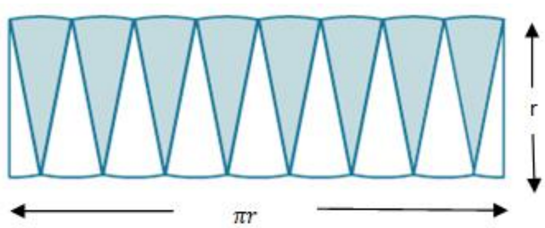
To find the formula for the area of a circle, you can cut a circle into 16 equal pieces.



Then you would arrange the triangular wedges by alternating the "triangle" directions and sliding them together to make a "parallelogram." Then you would cut the triangle on the left side in half on the given line, and slide the outside half of the triangle to the other end of the parallelogram in order to create an approximate "rectangle."







The circumference is $2\pi r$, where the radius is "r." Therefore, half of the circumference is πr.

What is the area of the "rectangle" using the side lengths above?

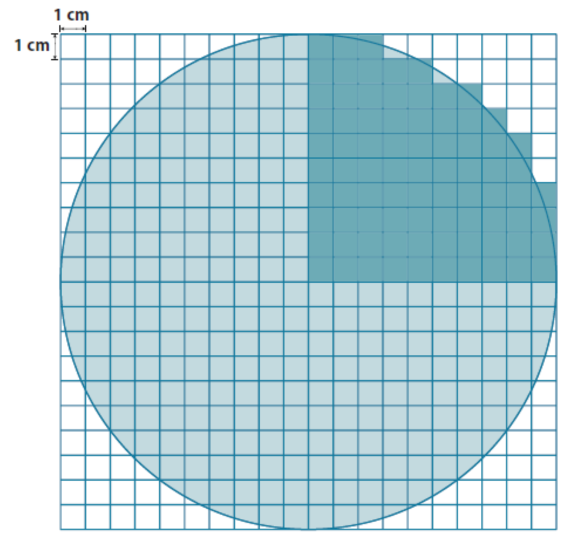
Are the areas of the rectangle and the circle the same?

If the area of the rectangular shape and the circle are the same, what is the area of the circle?

Example 1

Use the shaded square centimeter units to approximate the area of the circle.

What is the radius of the circle?



What would be a quicker method for determining the area of the circle other than counting all of the squares in the entire circle?

Using the diagram, how many squares were used to cover one-fourth of the circle?

What is the area of the entire circle?

Example 2

A sprinkler rotates in a circular pattern and sprays water over a distance of 12 feet. What is the area of the circular region covered by the sprinkler? Express your answer to the nearest square foot.

Draw a diagram to assist you in solving the problem. What does the distance of 12 feet represent in this problem?

What information is needed to solve the problem?

Example 3

Suzanne is making a circular table out of a square piece of wood. The radius of the circle that she is cutting is 3 feet. How much waste will she have for this project? Express your answer to the nearest square foot.

Draw a diagram to assist you in solving the problem. What does the distance of 3 feet represent in this problem?

What information is needed to solve the problem?

What information do we need to determine the area of the square and the circle?

How will we determine the waste?

Does your solution answer the problem as stated?

Exercises 4–6

4. A circle has a radius of 2 cm.
   a. Find the exact area of the circular region.

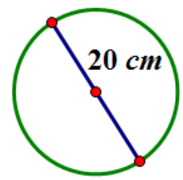
   b. Find the approximate area using 3.14 to approximate π.

5. A circle has a radius of 7 cm.
   a. Find the exact area of the circular region.

b. Find the approximate area using $\frac{22}{7}$ to approximate π.

c. What is the circumference of the circle?

6. Joan determined that the area of the circle below is $400\pi \text{ cm}^2$. Melinda says that Joan's solution is incorrect; she believes that the area is $100\pi \text{ cm}^2$. Who is correct and why?

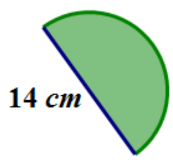


Lesson 18 (from Module 3): More Problems on Area and Circumference

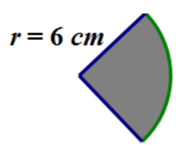
SEMICIRCLE: A semicircle is half of a circle. The area a semicircle can be found by finding the area of the full circle and dividing by 2.

Example 1

Find the area of the following semicircle. Use $\pi \approx \frac{22}{7}$.



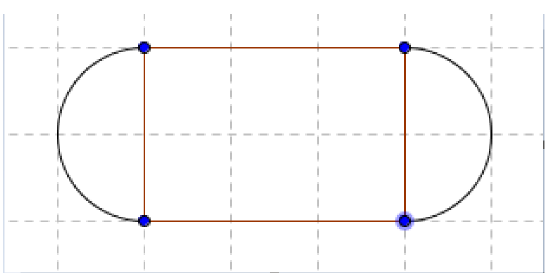
What is the area of the quarter circle?



Example 2

Marjorie is designing a new set of placemats for her dining room table. She sketched a drawing of the placement on graph paper. The diagram represents the area of the placemat consisting of a rectangle and two semicircles at either end. Each square on the grid measures 4 inches in length.

Find the area of the entire placemat. Explain your thinking regarding the solution to this problem.



If Marjorie wants to make six placemats, how many square inches of fabric will she need? Assume there is no waste.

Marjorie decides that she wants to sew on a contrasting band of material around the edge of the placemats. How much band material will Marjorie need?

Example 3

The circumference of a circle is 24π cm. What is the exact area of the circle?

Draw a diagram to assist you in solving the problem.

What information is needed to solve the problem?

Next, find the area.

Exercises

1. Find the area of a circle with a diameter of 42 cm. Use $\pi \approx \frac{22}{7}$.

2. The circumference of a circle is $9\pi\text{ cm}$.

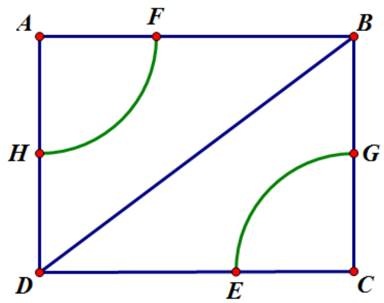
   a. What is the diameter?

   b. What is the radius?

   c. What is the area?

3. If students only know the radius of a circle, what other measures could they determine? Explain how students would use the radius to find the other parts.

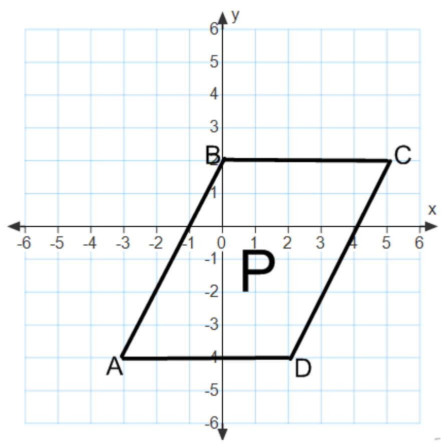
4. Find the area in the rectangle between the two quarter circles if $AF = 7$ ft., $FB = 9$ ft., and $HD = 7$ ft. Use $\pi \approx \frac{22}{7}$. Each quarter circle in the top-left and lower-right corners have the same radius.



Lesson 19 (from Module 3): Unknown Area Problems on the Coordinate Plane

Classwork

Example: Area of a Parallelogram



The coordinate plane below contains figure P, parallelogram $ABCD$.

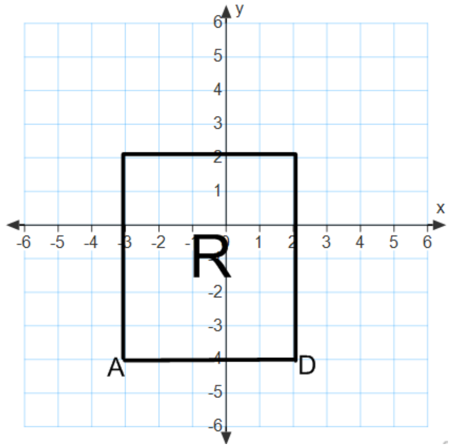
a. Write the ordered pairs of each of the vertices next to the vertex points.

b. Draw a rectangle surrounding figure P that has vertex points of A and C. Label the two triangles in the figure as S and T.

c. Find the area of the rectangle.

d. Find the area of each triangle.

e. Use these areas to find the area of parallelogram $ABCD$.



The coordinate plane below contains figure R, a rectangle with the same base as the parallelogram above.

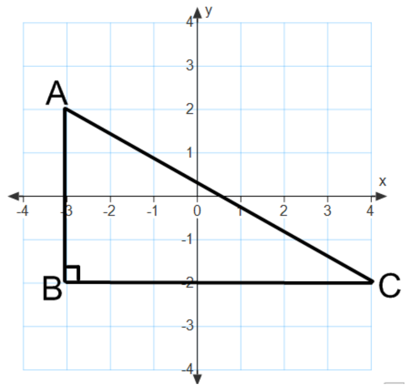
f. Draw triangles S and T and connect to figure R so that you create a rectangle that is the same size as the rectangle you created on the first coordinate plane.

g. Find the area of rectangle R.

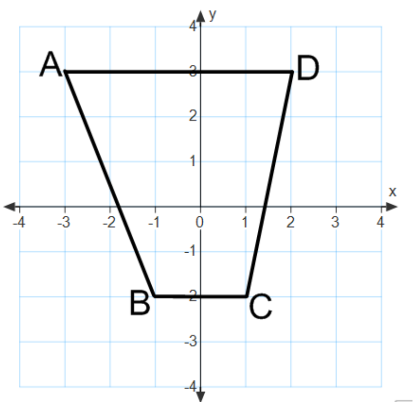
h. What do figures R and P have in common?

Exercises

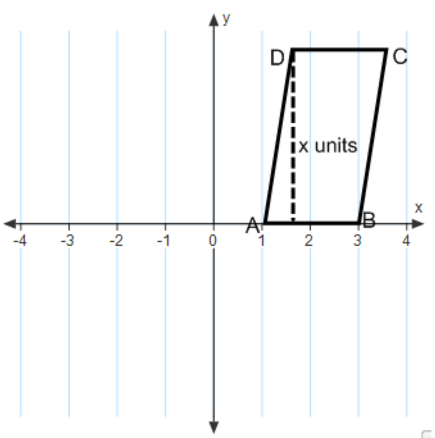
1. Find the area of triangle ABC.



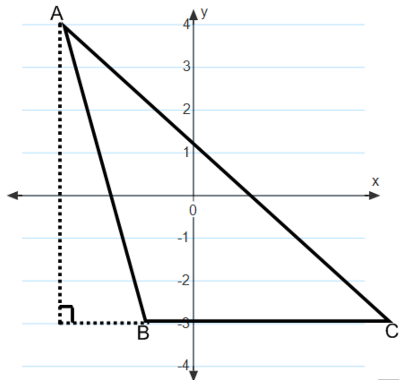
3. Find the area of quadrilateral $ABCD$ two different ways.



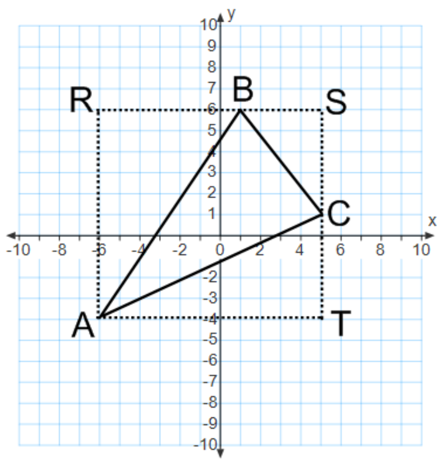
4. The area of quadrilateral $ABCD = 12$ sq. units. Find x.



5. The area of triangle $ABC = 14$ sq. units. Find the length of side BC.



6. Find the area of triangle ABC.

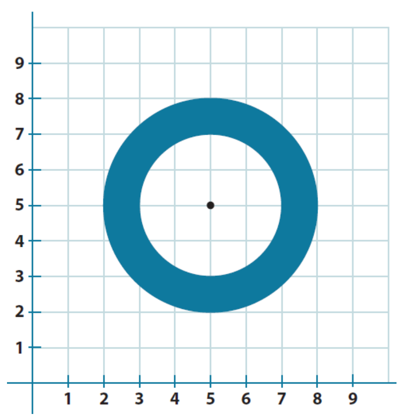


Lesson 20 (from Module 3): Composite Area Problems

Classwork

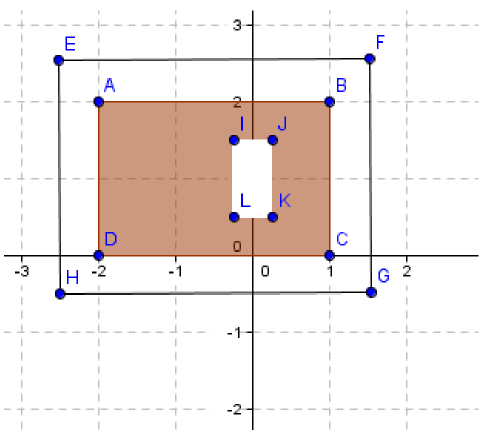
Example 1

Find the composite area of the shaded region. Use 3.14 for π.



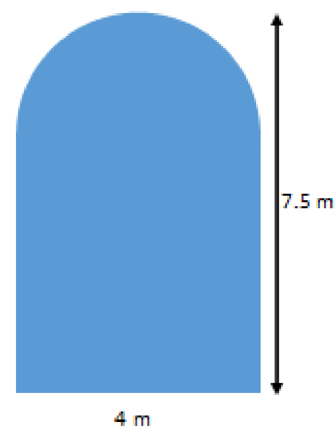
Exercise 1

A yard is shown with the shaded section indicating grassy areas and the unshaded sections indicating paved areas. Find the area of the space covered with grass in units^2.



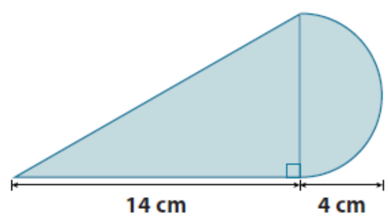
Example 2

Find the area of the figure that consists of a rectangle with a semicircle on top. Use 3.14 for π.



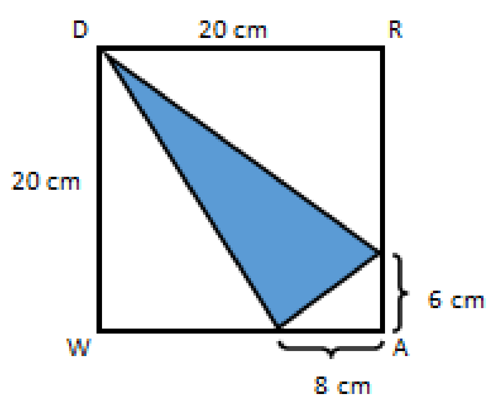
Exercise 2

Find the area of the shaded region. Use 3.14 for π.



Example 3

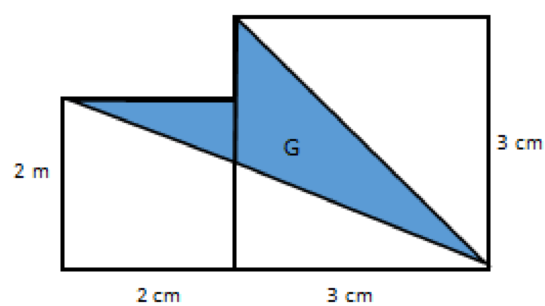
Find the area of the shaded region.



Redraw the figure separating the triangles; then, label the lengths discussing the calculations.

Exercise 3

Find the area of the shaded region. The figure is not drawn to scale.



Lesson 16: Slicing a Right Rectangular Prism with a Plane

Classwork

Example 1

Consider a ball B. Figure 3 shows one possible slice of B.

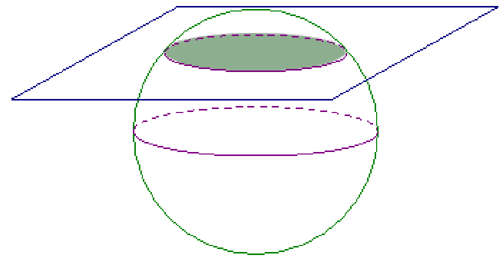


Figure 3. A Slice of Ball B

a. What figure does the slicing plane form? Students may choose their method of representation of the slice (e.g., drawing a 2D sketch, a 3D sketch, or describing the slice in words).

b. Will all slices that pass through B be the same size? Explain your reasoning.

c. How will the plane have to meet the ball so that the plane section consists of just one point?

Example 2

The right rectangular prism in Figure 4 has been sliced with a plane parallel to face $ABCD$. The resulting slice is a rectangular region that is identical to the parallel face.

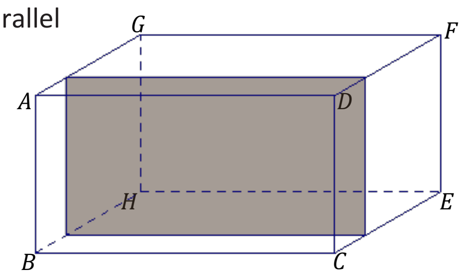


Figure 4.

a. Label the vertices of the rectangular region defined by the slice as $WXYZ$.

b. To which other face is the slice parallel and identical?

c. Based on what you know about right rectangular prisms, which faces must the slice be perpendicular to?

Exercise 1

Discuss the following questions with your group.

2. The right rectangular prism in Figure 5 has been sliced with a plane parallel to face $LMON$.

   a. Label the vertices of the rectangle defined by the slice as $RSTU$.

   b. What are the dimensions of the slice?

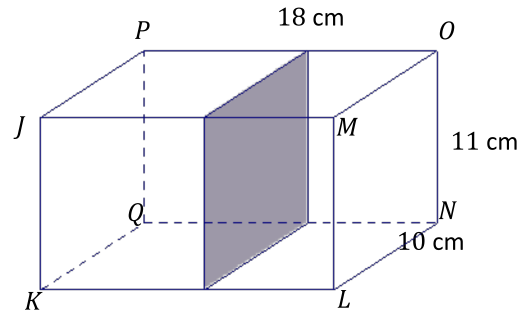


Figure 5.

   c. Based on what you know about right rectangular prisms, which faces must the slice be perpendicular to?

Example 3

The right rectangular prism in Figure 6 has been sliced with a plane perpendicular to $BCEH$. The resulting slice is a rectangular region with a height equal to the height of the prism.

a. Label the vertices of the rectangle defined by the slice as $WXYZ$.

b. To which other face is the slice perpendicular?

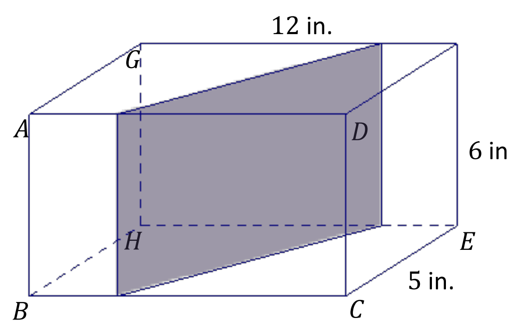


Figure 6.

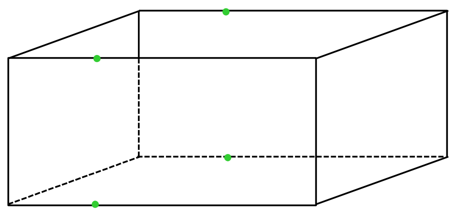
c. What is the length of ZY?

d. Joey looks at $WXYZ$ and thinks that the slice may be a parallelogram that is not a rectangle. Based on what is known about how the slice is made, can he be right? Justify your reasoning.

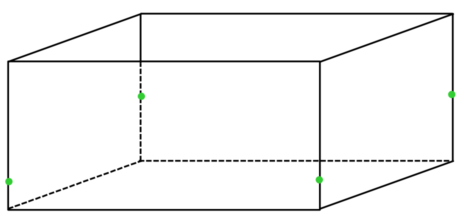
Exercises 2–6

In the following exercises, the points at which a slicing plane meets the edges of the right rectangular prism have been marked. Each slice is either parallel or perpendicular to a face of the prism. Use a straightedge to join the points to outline the rectangular region defined by the slice and shade in the rectangular slice.

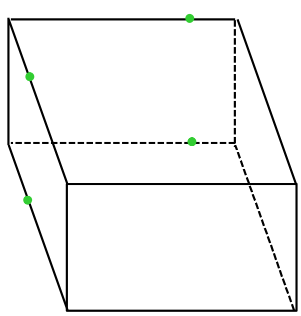
2. A slice parallel to a face



3. A slice perpendicular to a face.

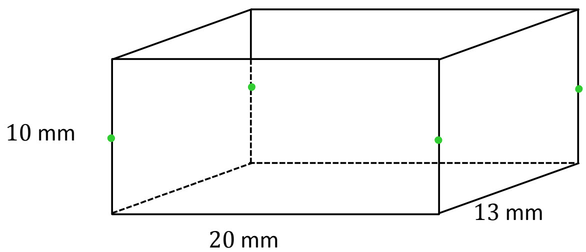


4. A slice perpendicular to a face.

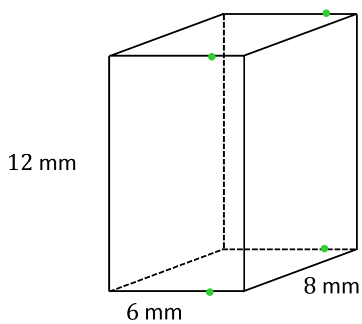


In Exercises 5–6, the dimensions of the prisms have been provided. Use the dimensions to sketch the slice from each prism and provide the dimensions of each slice.

5. A slice parallel to a face.



6. A slice perpendicular to a face.

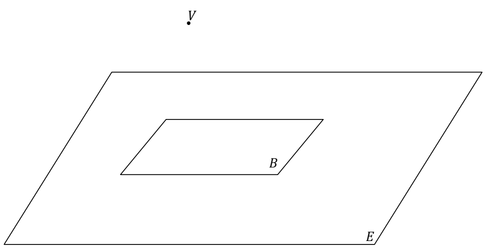


Lesson 17: Slicing a Right Rectangular Pyramid with a Plane

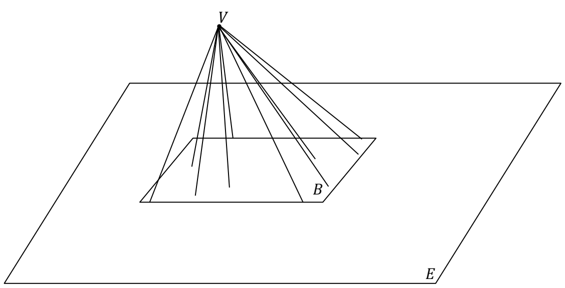
Classwork

Opening

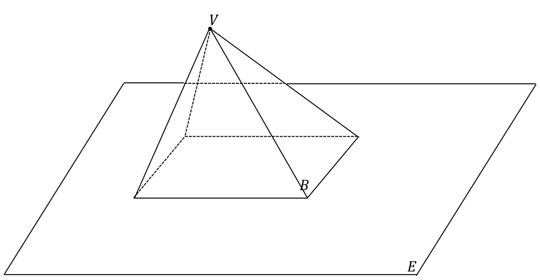
<u>Rectangular Pyramid</u>: Given a rectangular region B in a plane E, and a point V not in E, the *rectangular pyramid with base B and vertex V* is the collection of all segments $\overline{VP}$ for any point P in B. It can be shown that the planar region defined by a side of the base B and the vertex V is a triangular region, called a *lateral face*.



A rectangular region B in a plane E and a point V not in E

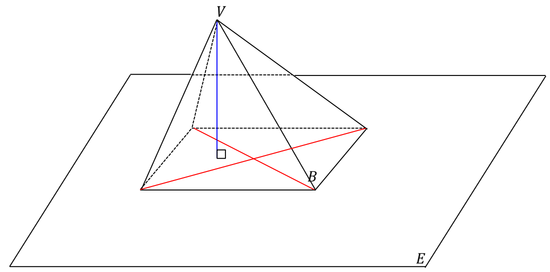


The rectangular pyramid will be determined by the collection of all segments $\overline{VP}$ for any point P in B; here $\overline{VP}$ is shown for a total of 10 points.

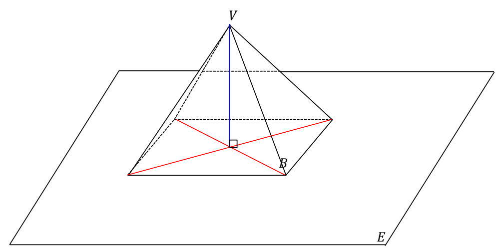


The rectangular pyramid is a solid once the collection of all segments $\overline{VP}$ for any point P in B are taken. The pyramid has a total of five faces: four lateral faces and a base.

If the vertex lies on the line perpendicular to the base at its center (the intersection of the rectangle's diagonals), the pyramid is called a right rectangular pyramid. The example of the rectangular pyramid above is not a right rectangular pyramid, as evidenced in this image. The perpendicular from V does not meet at the intersection of the diagonals of the rectangular base B.



The following is an example of a right rectangular pyramid. The opposite lateral faces are identical isosceles triangles.



Example 1

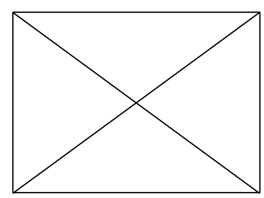
Use the models you built to assist in a sketch of a pyramid: Though you are sketching from a model that is opaque, use dotted lines to represent the edges that cannot be seen from your perspective.

Example 2

Sketch a right rectangular pyramid from three vantage points: (1) from directly over the vertex, (2) facing straight on to a lateral face, and (3) from the bottom of the pyramid. Explain how each drawing shows each view of the pyramid.

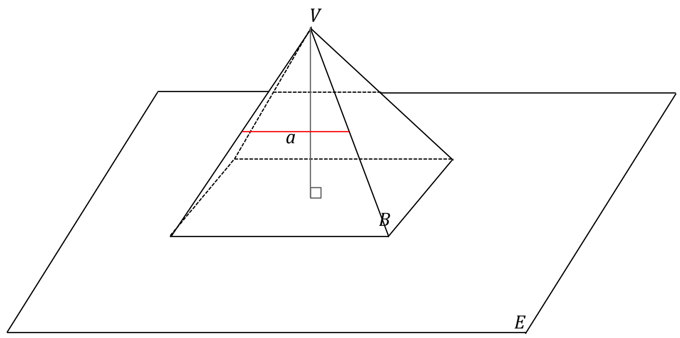
Example 3

Assume the following figure is a top-down view of a rectangular pyramid. Make a reasonable sketch of any two adjacent lateral faces. What measurements must be the same between the two lateral faces? Mark the equal measurement. Justify your reasoning for your choice of equal measurements.



Example 4

a. A slicing plane passes through segment a parallel to base B of the right rectangular pyramid below. Sketch what the slice will look like into the figure. Then sketch the resulting slice as a two-dimensional figure. Students may choose how to represent the slice (e.g., drawing a 2D or 3D sketch or describing the slice in words).

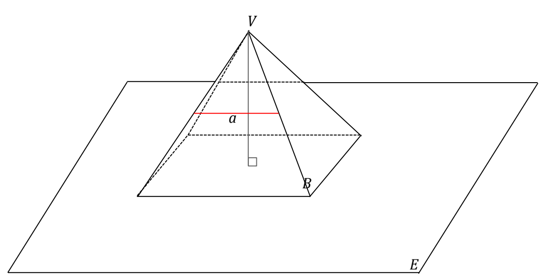


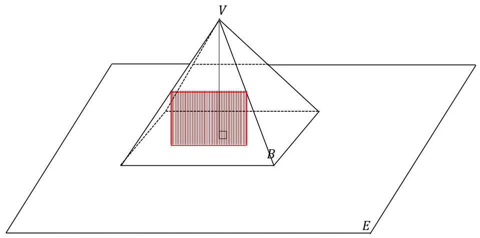
b. What shape does the slice make? What is the relationship between the slice and the rectangular base of the pyramid?

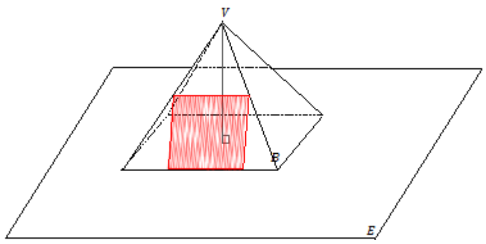
Example 5

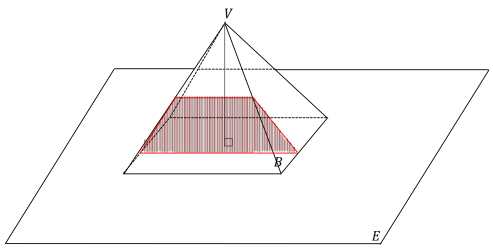
A slice is to be made along segment a perpendicular to base B of the right rectangular pyramid below.

a. Which of the following figures shows the correct slice? Justify why each of the following figures is or is not a correct diagram of the slice.

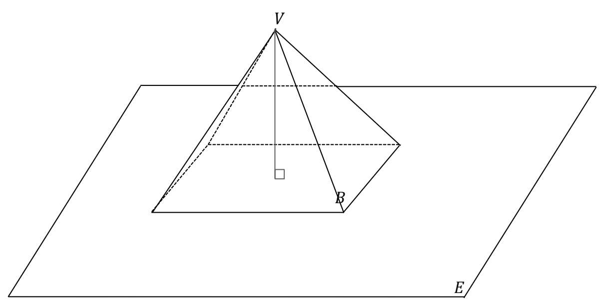








b. A slice is taken through the vertex of the pyramid perpendicular to the base. Sketch what the slice will look like into the figure. Then, sketch the resulting slice itself as a two-dimensional figure.

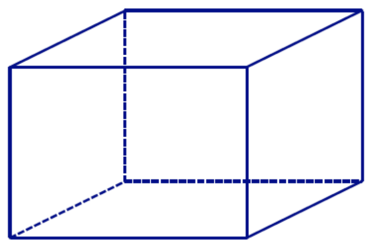


Lesson 18: Slicing on an Angle

Classwork

Example 1

With your group, discuss whether a right rectangular prism can be sliced at an angle so that the resulting slice looks like the figure in Figure 1? If it is possible, draw an example of such a slice into the following prism.



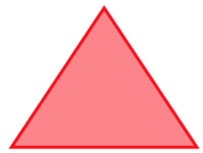


Figure 1

Exercise 1

a. With your group, discuss how to slice a right rectangular prism so that the resulting slice looks like the figure in Figure 2. Justify your reasoning.

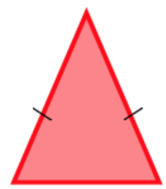


Figure 2

b. With your group, discuss how to slice a right rectangular prism so that the resulting slice looks like the figure in Figure 3. Justify your reasoning.

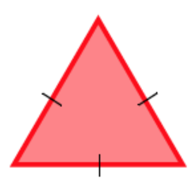


Figure 3

Example 2

With your group, discuss whether a right rectangular prism can be sliced at an angle so that the resulting slice looks like the figure in Figure 4. If it is possible, draw an example of such a slice into the following prism.

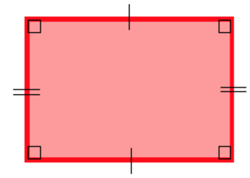
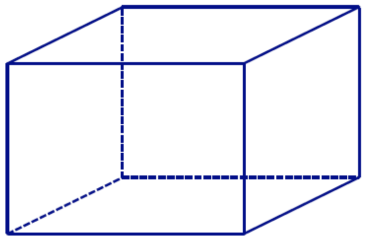


Figure 4



Exercise 2

In Example 2, we discovered how to slice a right rectangular prism to makes the shapes of a rectangle and a parallelogram. Are there other ways to slice a right rectangular prism that result in other quadrilateral-shaped slices?

Example 3

a. Slicing a plane through a right rectangular prism so that the slice meets the three faces of the prism, the resulting slice is in the shape of a triangle; if the slice meets four faces, the resulting slice is in the shape of a quadrilateral. Is it possible to slice the prism in a way that the region formed is a pentagon (as in Figure 5)? A hexagon (as in Figure 6)? An octagon (as in Figure 7)?

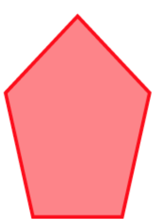


Figure 5

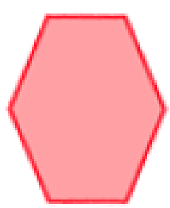


Figure 6

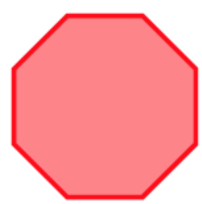
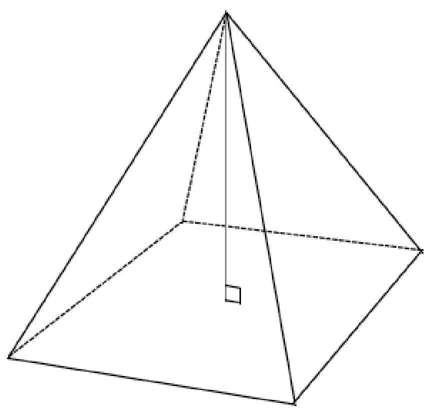


Figure 7

b. Draw an example of a slice in a pentagon shape and a slice in a hexagon shape.

Example 4

a. With your group, discuss whether a right rectangular pyramid can be sliced at an angle so that the resulting slice looks like the figure in Figure 8. If it is possible, draw an example of such a slice into the following pyramid.



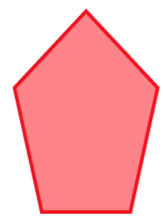


Figure 8

b. With your group, discuss whether a right rectangular pyramid can be sliced at an angle so that the resulting slice looks like the figure in Figure 9. If it is possible, draw an example of such a slice into the pyramid above.

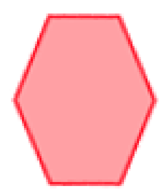


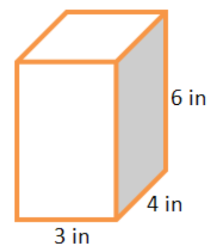
Figure 9

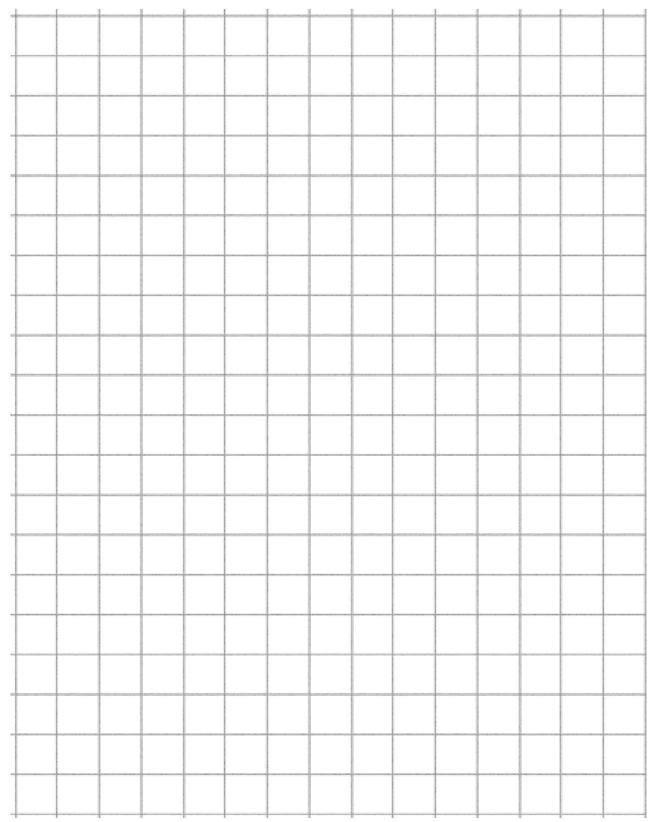
Lesson 21 (from Module 3): Surface Area

Classwork

Opening Exercise: Surface Area of a Right Rectangular Prism

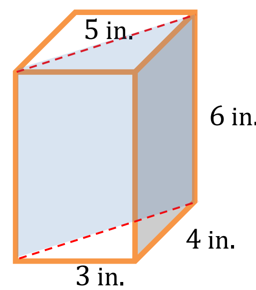
On the provided grid, draw a net representing the surfaces of the right rectangular prism (assume each grid line represents 1 inch). Then, find the surface area of the prism by finding the area of the net.

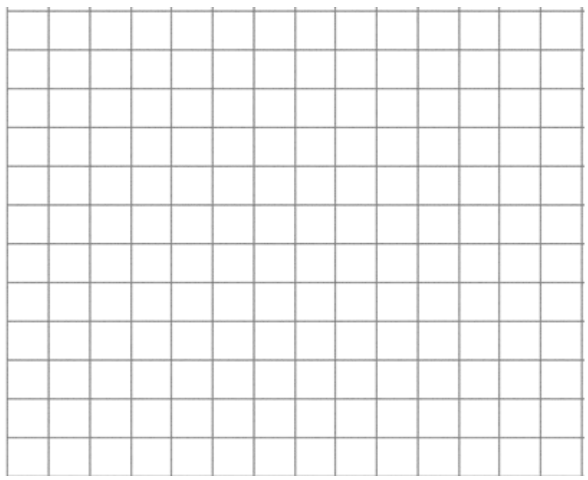




Exercise 1

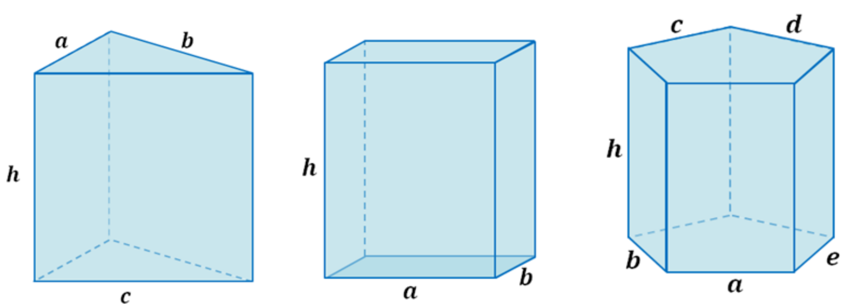
Marcus thinks that the surface area of the right triangular prism will be half that of the right rectangular prism and wants to use the modified formula $SA = \frac{1}{2}(2lw + 2lh + 2wh)$. Do you agree or disagree with Marcus? Use nets of the prisms to support your argument.





Example 1: Lateral Area of a Right Prism

A right triangular prism, a right rectangular prism, and a right pentagonal prism are pictured below, and all have equal heights of h.



a. Write an expression that represents the lateral area of the right triangular prism as the sum of the areas of its lateral faces.

b. Write an expression that represents the lateral area of the right rectangular prism as the sum of the areas of its lateral faces.

c. Write an expression that represents the lateral area of the right pentagonal prism as the sum of the areas of its lateral faces.

d. What value appears often in each expression and why?

e. Rewrite each expression in factored form using the distributive property and the height of each lateral face.

f. What do the parentheses in each case represent with respect to the right prisms?

g. How can we generalize the lateral area of a right prism into a formula that applies to all right prisms?

1. Why are the lateral faces of right prisms always rectangular regions?

2. What is the name of the right prism whose bases are rectangles?

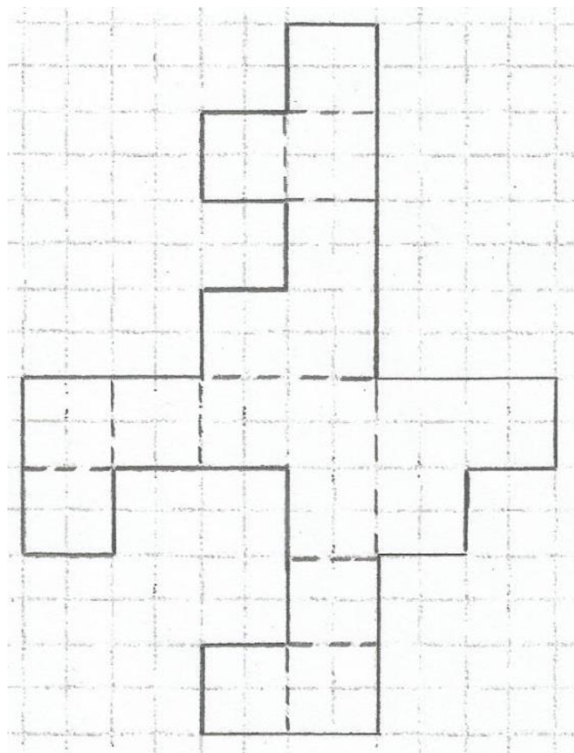
3. How does this definition of right prism include the interior of the prism?

Lesson 22 (from Module 3): Surface Area

Classwork

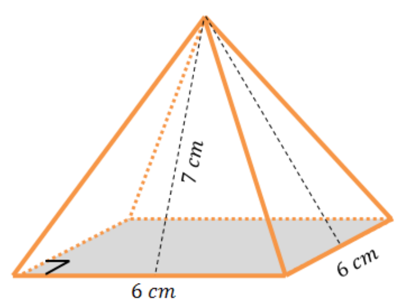
Opening Exercise

What is the area of the composite figure in the diagram? Is the diagram a net for a three-dimensional image? If so, sketch the image. If not, explain why.



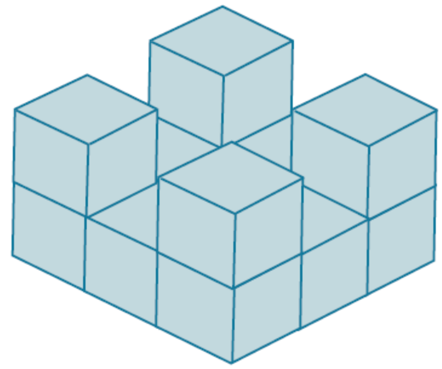
Example 1

The pyramid in the picture has a square base, and its lateral faces are triangles that are exact copies of one another. Find the surface area of the pyramid.



Example 2: Using Cube

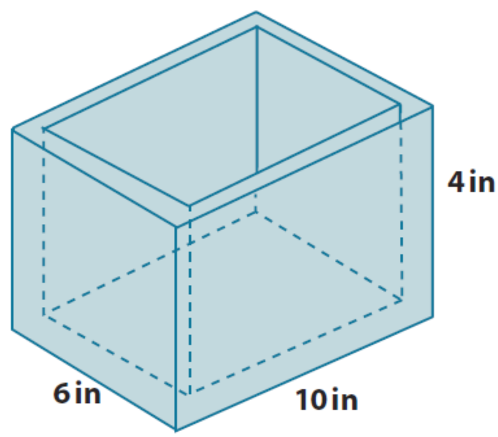
There are 13 cubes glued together forming the solid in the diagram. The edges of each cube are $\frac{1}{4}$ inch in length. Find the surface area of the solid.



Example 3

Find the total surface area of the wooden jewelry box. The sides and bottom of the box are all $\frac{1}{4}$ inch thick.

What are the faces that make up this box?



How does this box compare to other objects that you have found the surface area of?

Large Prism

Small Prism

Surface Area of the Box

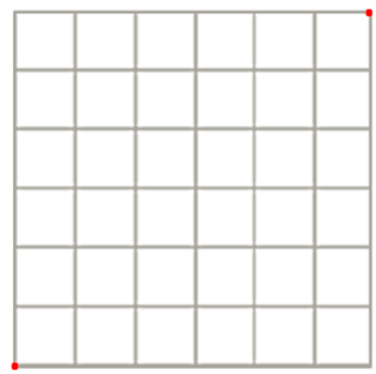
Lesson 23 (from Module 3): The Volume of a Right Prism

Classwork

Opening Exercise

The volume of a solid is a quantity given by the number of unit cubes needed to fill the solid. Most solids—rocks, baseballs, people—cannot be filled with unit cubes or assembled from cubes. Yet such solids still have volume. Fortunately, we do not need to assemble solids from unit cubes in order to calculate their volume. One of the first interesting examples of a solid that cannot be assembled from cubes but whose volume can still be calculated from a formula is a right triangular prism.

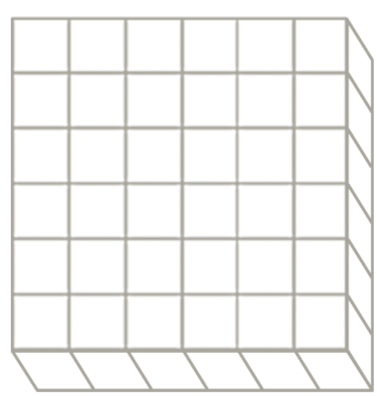
What is the area of the square pictured on the right? Explain.



Draw the diagonal joining the two given points; then, darken the grid lines within the lower triangular region. What is area of that triangular region? Explain.

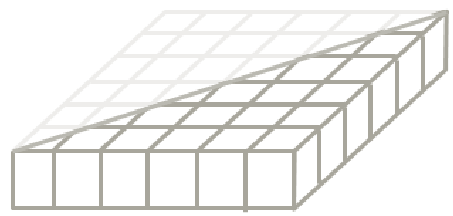
Exploratory Challenge: The Volume of a Right Prism

What is the volume of the right prism pictured on the right? Explain.

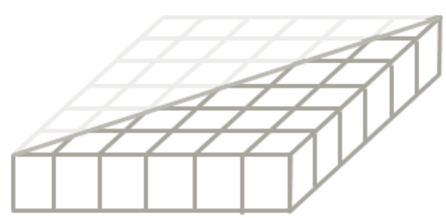


Draw the same diagonal on the square base as done above; then, darken the grid lines on the lower right triangular prism. What is the volume of that right triangular prism? Explain.

How could we create a right triangular prism with five times the volume of the right triangular prism pictured to the right, without changing the base? Draw your solution on the diagram, give the volume of the solid, and explain why your solution has five times the volume of the triangular prism.



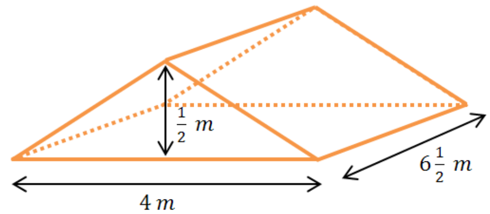
What could we do to cut the volume of the right triangular prism pictured on the right in half without changing the base? Draw your solution on the diagram, give the volume of the solid, and explain why your solution has half the volume of the given triangular prism.



To find the volume (V) of any right prism …

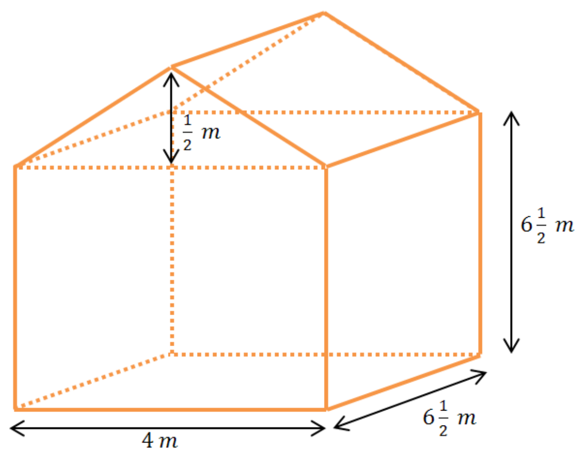
Example: The Volume of a Right Triangular Prism

Find the volume of the right triangular prism shown in the diagram using $V = Bh$.



Exercise: Multiple Volume Representations

The right pentagonal prism is composed of a right rectangular prism joined with a right triangular prism. Find the volume of the right pentagonal prism shown in the diagram using two different strategies.

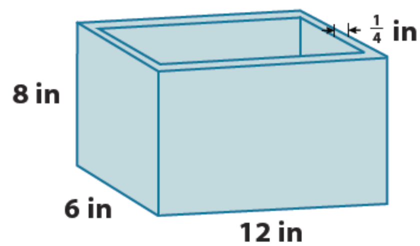


Lesson 24 (from Module 3): The Volume of a Right Prism

Classwork

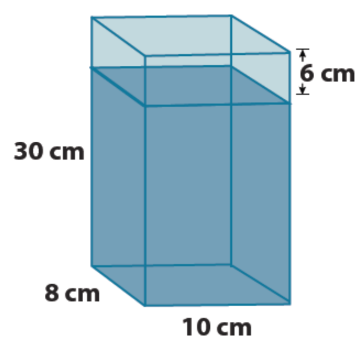
Exploratory Challenge: Measuring a Container's Capacity

A box in the shape of a right rectangular prism has a length of 12 in., a width of 6 in., and a height of 8 in. The base and the walls of the container are $\frac{1}{4}$ in. thick, and its top is open. What is the capacity of the right rectangular prism?
(Hint: The capacity is equal to the volume of water needed to fill the prism to the top.)



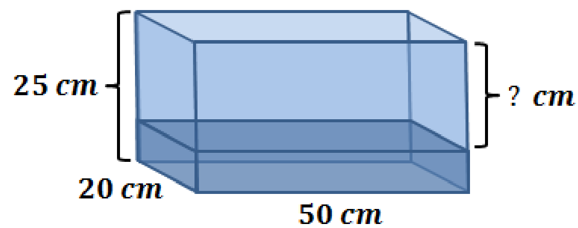
Example 1: Measuring Liquid in a Container in Three Dimensions

A glass container is in the form of a right rectangular prism. The container is 10 cm long, 8 cm wide, and 30 cm high. The top of the container is open, and the base and walls of the container are 3 mm (or 0.3 cm) thick. The water in the container is 6 cm from the top of the container. What is the volume of the water in the container?



Example 2

7.2 L of water are poured into a container in the shape of a right rectangular prism. The inside of the container is 50 cm long, 20 cm wide, and 25 cm tall. How far from the top of the container is the surface of the water? ($1 \text{ L} = 1000 \text{ cm}^3$)



Example 3

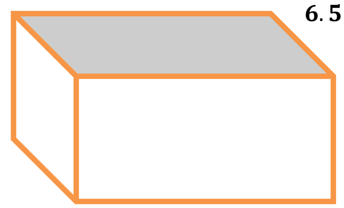
A fuel tank is the shape of a right rectangular prism and has 27 L of fuel in it. It is determined that the tank is $\frac{3}{4}$ full. The inside dimensions of the base of the tank are 90 cm by 50 cm. What is the height of the fuel in the tank? How deep is the tank? ($1 \text{ L} = 1000 \text{ cm}^3$)

Lesson 25 (from Module 3): Volume and Surface Area

Classwork

Opening Exercise

What is the surface area and volume of the right rectangular prism?



Example 1: Volume of a Fish Tank

Jay has a small fish tank. It is the same shape and size as the right rectangular prism shown in the Opening Exercise.

a. The box it came in says that it is a 3 gallon tank. Is this claim true? Explain your reasoning. Recall that
$1 \text{ gal} = 231 \text{ in}^3$.

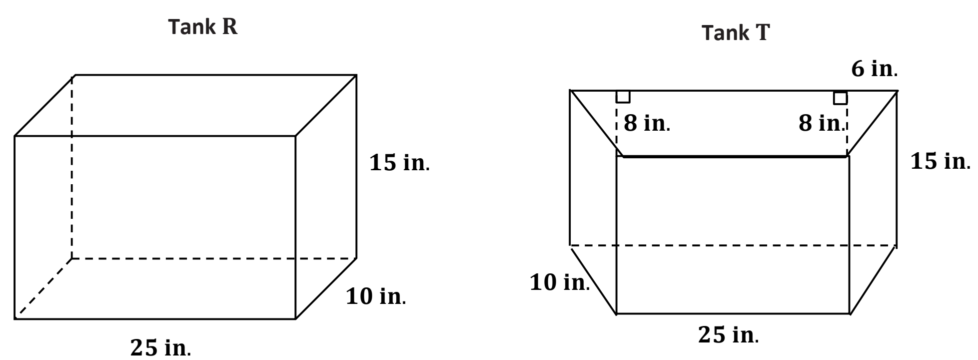
b. The pet store recommends filling the tank to within 1.5 in. of the top. How many gallons of water will the tank hold if it is filled to the recommended level?

c. Jay wants to cover the back, left, and right sides of the tank with a background picture. How many square inches will be covered by the picture?

d. Water in the tank evaporates each day, causing the water level to drop. How many gallons of water have evaporated by the time the water in the tank is four inches deep? Assume the tank was filled to within 1.5 in. of the top to start.

Exercise 1: Fish Tank Designs

Two fish tanks are shown below, one in the shape of a right rectangular prism (R) and one in the shape of a right trapezoidal prism (T).



a. Which tank holds the most water? Let $Vol(R)$ represent the volume of the right rectangular prism and $Vol(T)$ represent the volume of the right trapezoidal prism. Use your answer to fill in the blanks with $Vol(R)$ and $Vol(T)$.

$$\_\_\_\_\_\_\_\_\_\_\_\_ < \_\_\_\_\_\_\_\_\_\_\_\_$$

b. Which tank has the most surface area? Let $SA(R)$ represent the surface area of the right rectangular prism and $SA(T)$ represent the surface area of the right trapezoidal prism. Use your answer to fill in the blanks with $SA(R)$ and $SA(T)$.

_________________ $<$ _________________

c. Water evaporates from each aquarium. After the water level has dropped $\frac{1}{2}$ inch in each aquarium, how many cubic inches of water are required to fill up each aquarium? Show work to support your answers.

Exercise 2: Design Your Own Fish Tank

Design at least three fish tanks that will hold approximately 10 gallons of water. All of the tanks should be shaped like right prisms. Make at least one tank have a base that is not a rectangle. For each tank, make a sketch, and calculate the volume in gallons to the nearest hundredth.

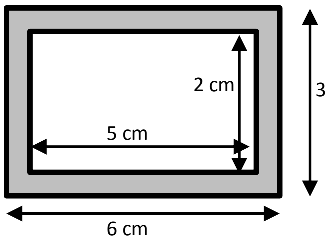
Challenge: Each tank is to be constructed from glass that is $\frac{1}{4}$ in. thick. Select one tank that you designed and determine the difference between the volume of the total tank (including the glass) and the volume inside the tank. Do not include a glass top on your tank.

Lesson 26 (from Module 3): Volume and Surface Area

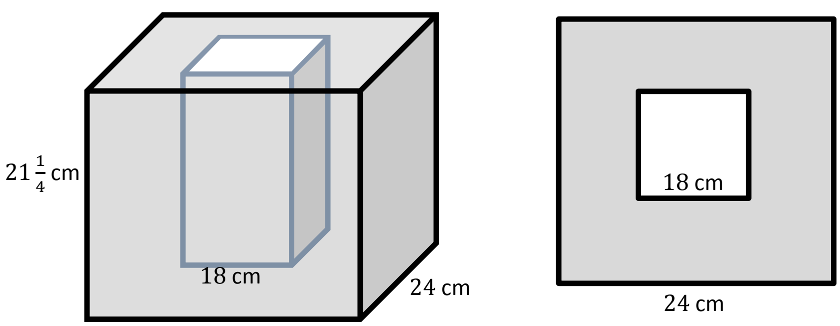
Classwork

Opening Exercise

Explain to your partner how you would calculate the area of the shaded region. Then, calculate the area.



Example 1: Volume of a Shell



The insulated box shown is made from a large cube with a hollow inside that is a right rectangular prism with a square base. The figure on the right is what the box looks like from above.

a. Calculate the volume of the outer box.

b. Calculate the volume of the inner prism.

c. Describe in words how you would find the volume of the insulation.

d. Calculate the volume of the insulation in cubic centimeters.

e. Calculate the amount of water the box can hold in liters.

Exercise 1: Designing a Brick Planter

You have been asked by your school to design a brick planter that will be used by classes to plant flowers. The planter will be built in the shape of a right rectangular prism with no bottom so water and roots can access the ground beneath. The exterior dimensions are to be $12 \text{ ft.} \times 9 \text{ ft.} \times 2\frac{1}{2} \text{ ft.}$ The bricks used to construct the planter are 6 in. long, $3\frac{1}{2} \text{ in.}$ wide, and 2 in. high.

a. What are the interior dimensions of the planter if the thickness of the planter's walls is equal to the length of the bricks?

b. What is the volume of the bricks that form the planter?

c. If you are going to fill the planter $\frac{3}{4}$ full of soil, how much soil will you need to purchase, and what will be the height of the soil?

d. How many bricks are needed to construct the planter?

e. Each brick used in this project costs $\$0.82$ and weighs 4.5 lb. The supply company charges a delivery fee of $\$15$ per whole ton (2000 lb.) over 4000 lb. How much will your school pay for the bricks (including delivery) to construct the planter?

f. A cubic foot of topsoil weighs between 75 and 100 lb. How much will the soil in the planter weigh?

g. If the topsoil costs $\$0.88$ per each cubic foot, calculate the total cost of materials that will be used to construct the planter.

Exercise 2: Design a Feeder

You did such a good job designing the planter that a local farmer has asked you to design a feeder for the animals on his farm. Your feeder must be able to contain at least $100{,}000$ cubic centimeters, but not more than $200{,}000$ cubic centimeters of grain when it is full. The feeder is to be built of stainless steel and must be in the shape of a right prism, but not a right rectangular prism. Sketch your design below including dimensions. Calculate the volume of grain that it can hold and the amount of metal needed to construct the feeder.

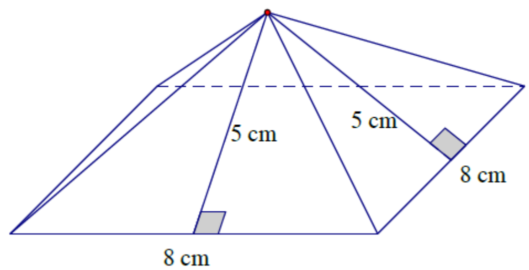
The farmer needs a cost estimate. Calculate the cost of constructing the feeder if $\frac{1}{2}$ cm thick stainless steel sells for $\$93.25$ per square meter.

Lesson 23: Surface Area

Classwork

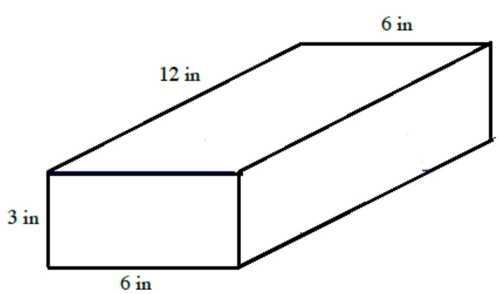
Opening Exercise

Calculate the surface area of the square pyramid.

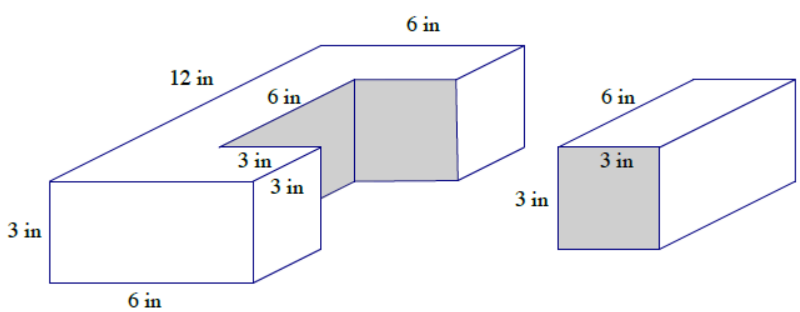


Example 1

a. Calculate the surface area of the rectangular prism.



b. Imagine that a piece of the rectangular prism is removed. Determine the surface area of both pieces.

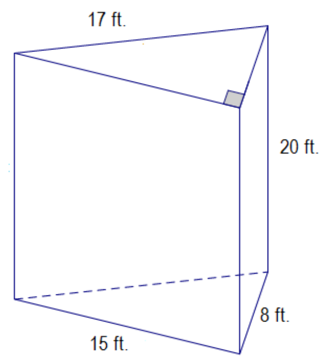


c. How is the surface area in part (a) related to the surface area in part (b)?

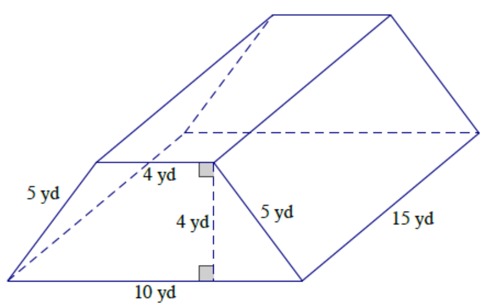
Exercises 1–5

Determine the surface area of the right prisms.

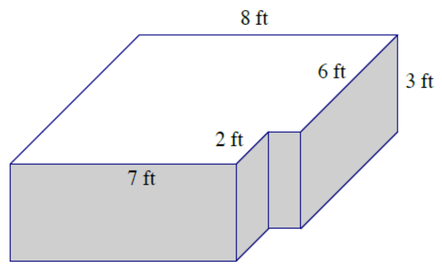
1.



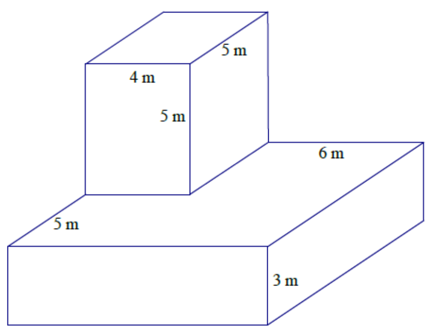
3.



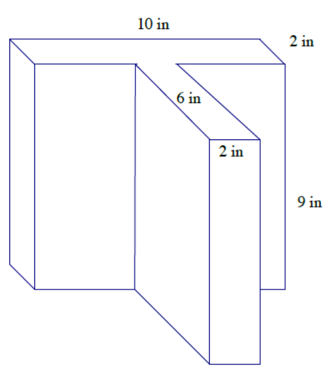
4.



5.



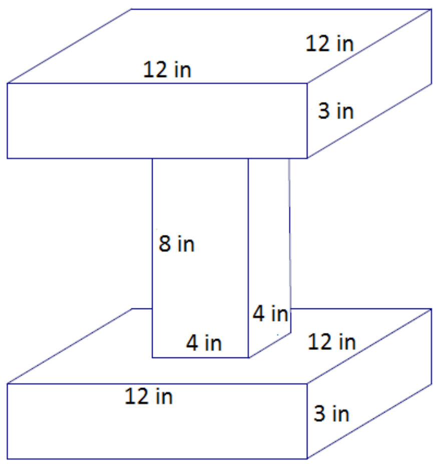
6.



Lesson 24: Surface Area

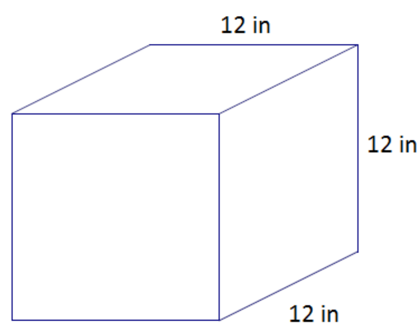
Classwork

Example 1

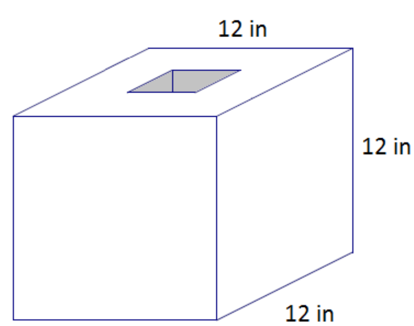


Example 2

a. Determine the surface area of the cube.



b. A square hole with a side length of 4 inches is drilled through the cube. Determine the new surface area.

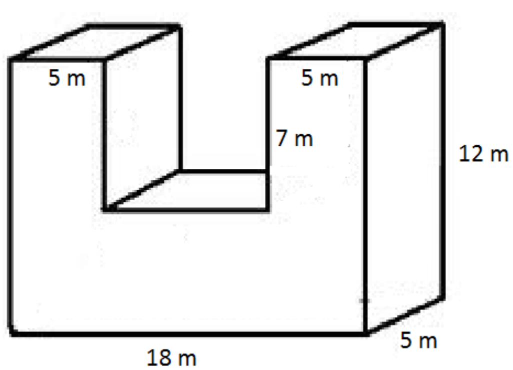


Example 3

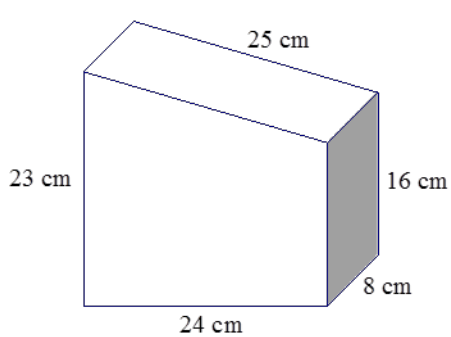
A right rectangular pyramid has a square base with a side length of 10 inches. The surface area of the pyramid is 260 in^2. Find the height of the four lateral triangular faces.

Exercises 1–8

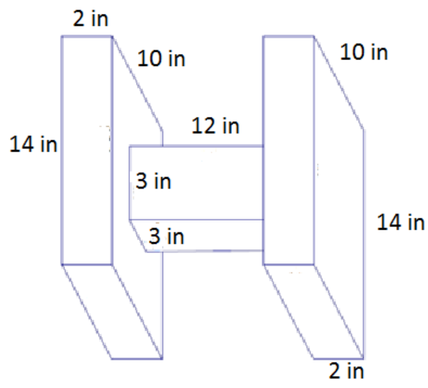
1. Determine the surface area of each figure. Assume all faces are rectangles unless it is indicated otherwise.



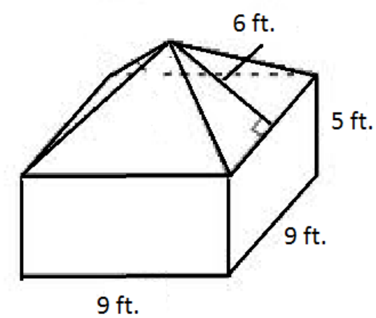
2. In addition to your calculation, explain how the surface area was determined.



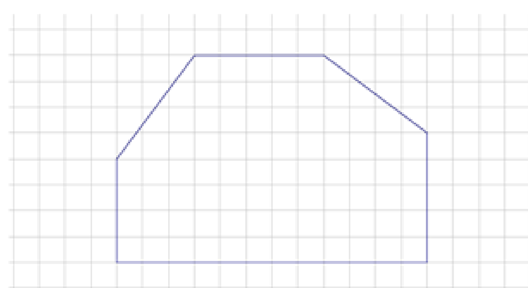
3.



4. In addition to your calculation, explain how the surface area was determined.

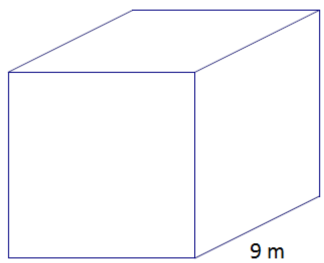


5. A hexagonal prism has the following base and has a height of 8 units. Determine the surface area of the prism.

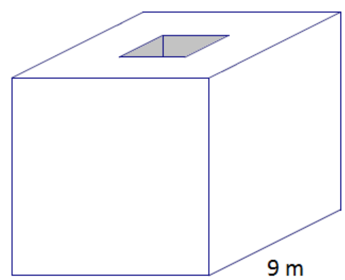


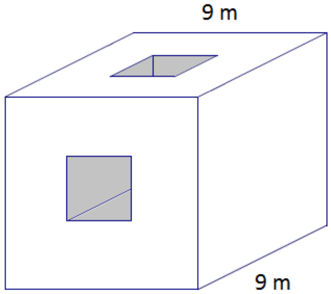
6. Determine the surface area of each figure.

   a.



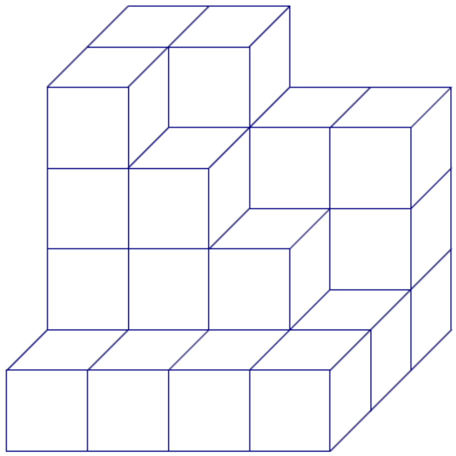
   b. A cube with a square hole with 3 m side lengths has been drilled through the cube.





   c. A second square hole with 3 m side lengths has been drilled through the cube.

7. The figure below shows 28 cubes with an edge length of 1 unit. Determine the surface area.



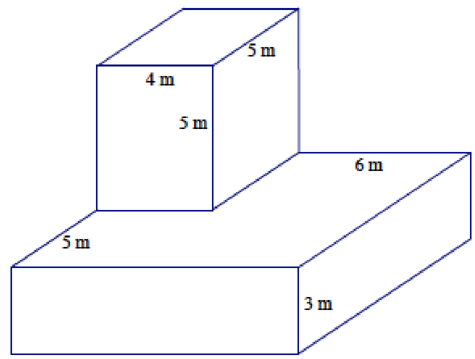
8. The base rectangle of a right rectangular prism is 4 ft. $\times$ 6 ft. The surface area is 288 ft^2. Find the height. Let h be the height in feet.

Lesson 26: Volume of Composite Three-Dimensional Objects

Classwork

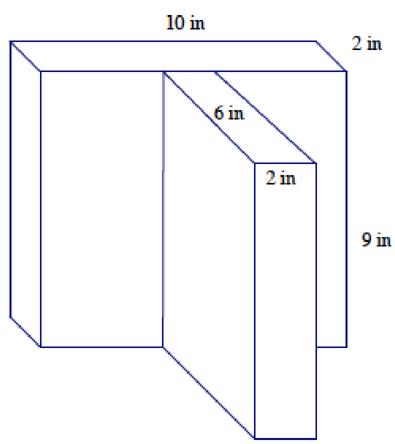
Example 1

1. Find the volume of the following three-dimensional object composed of two right rectangular prisms.



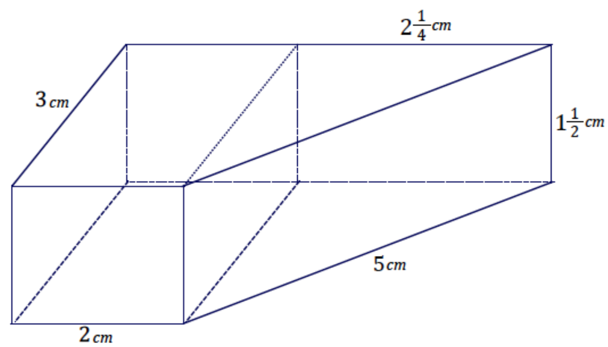
Exercise 1

Find the volume of the following three-dimensional figure composed of two right rectangular prisms.



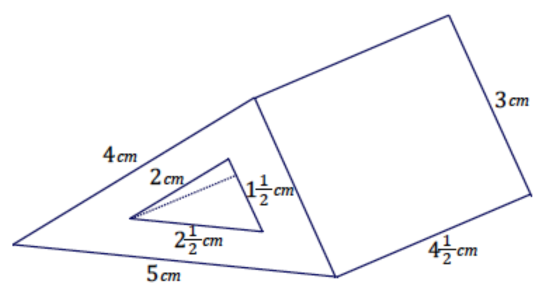
Exercise 2

The right trapezoidal prism is composed of a right rectangular prism joined with a right triangular prism. Find the volume of the right trapezoidal prism shown in the diagram using two different strategies.



Example 2

Find the volume of the right prism shown in the diagram whose base is the region between two right triangles. Use two different strategies.



Example 3

A box with a length of 2 ft., a width of 1.5 ft., and a height of 1.25 ft. contains fragile electronic equipment that is packed inside a larger box with three inches of styrofoam cushioning material on each side (above, below, left side, right side, front, and back).

a. Give the dimensions of the larger box.

b. Design styrofoam right rectangular prisms that could be placed around the box to provide the cushioning; i.e., give the dimensions and how many of each size are needed.

c. Find the volume of the styrofoam cushioning material by adding the volumes of the right rectangular prisms in the previous question.

d. Find the volume of the styrofoam cushioning material by computing the difference between the volume of the larger box and the volume of the smaller box.